

Getting to the Core

Algebra II

A9 – Functions – Unit of Study

Updated on May 3, 2013

Student Name _____ Period _____

This page was intentionally left blank.

Unit A9 – Functions

Table of Contents

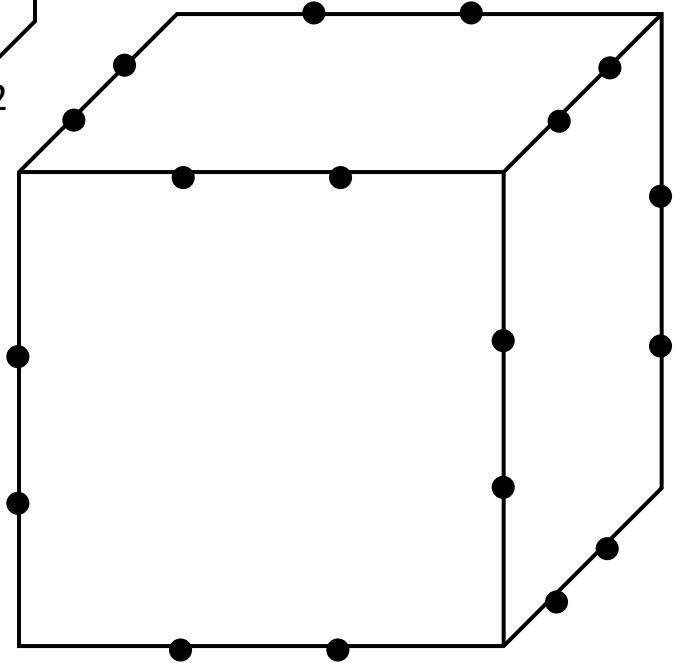
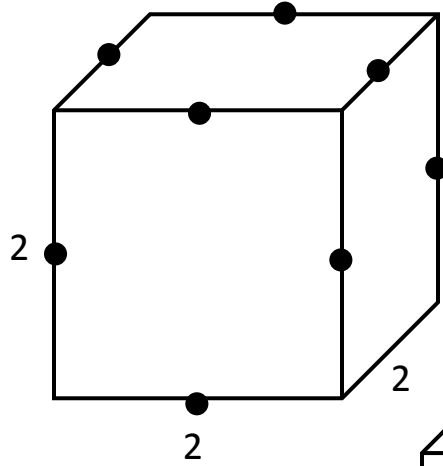
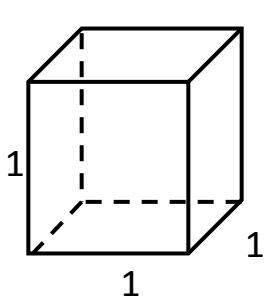
Lessons	Description	Page
Title page	Unit A9 – Functions	1
Table of Contents	Table of Contents	3
A9-H-1	Hook	
	Cube Growth	5
A9-PTL-A	Preparing the Learner Lesson	
	Functions Unit Pre-Assessment	9
	Functions Unit Skill Preview	10
A9-1-1	Multiple Representations: Expert Investigation	
	Growth Functions	11
	Summary of Key Features from Each of Five Representations	37
A9-2-1	Domain & Range	
	Domain & Range Texts	39
	Three Quadratic Functions	41
	Three Cubic Functions	42
	Three Linear Functions	43
	Three Exponential Functions	44
	Three Radical (Square Root) Functions	45
A9-SA-1	Summative Assessment Part 1	
	Growth Rates	47
A9-SA-2	Summative Assessment Part 2	
	Functions Assessment	49

This page was intentionally left blank.

Cube Growth

Name _____ Period __

Jason is creating cubes with different side lengths out of straws and construction paper. He needs your help to figure out how many straws (each straw is 1 ft) and sheets of paper (1ft by 1ft) he needs. Help Jason calculate how many materials he should purchase by completing the following table.

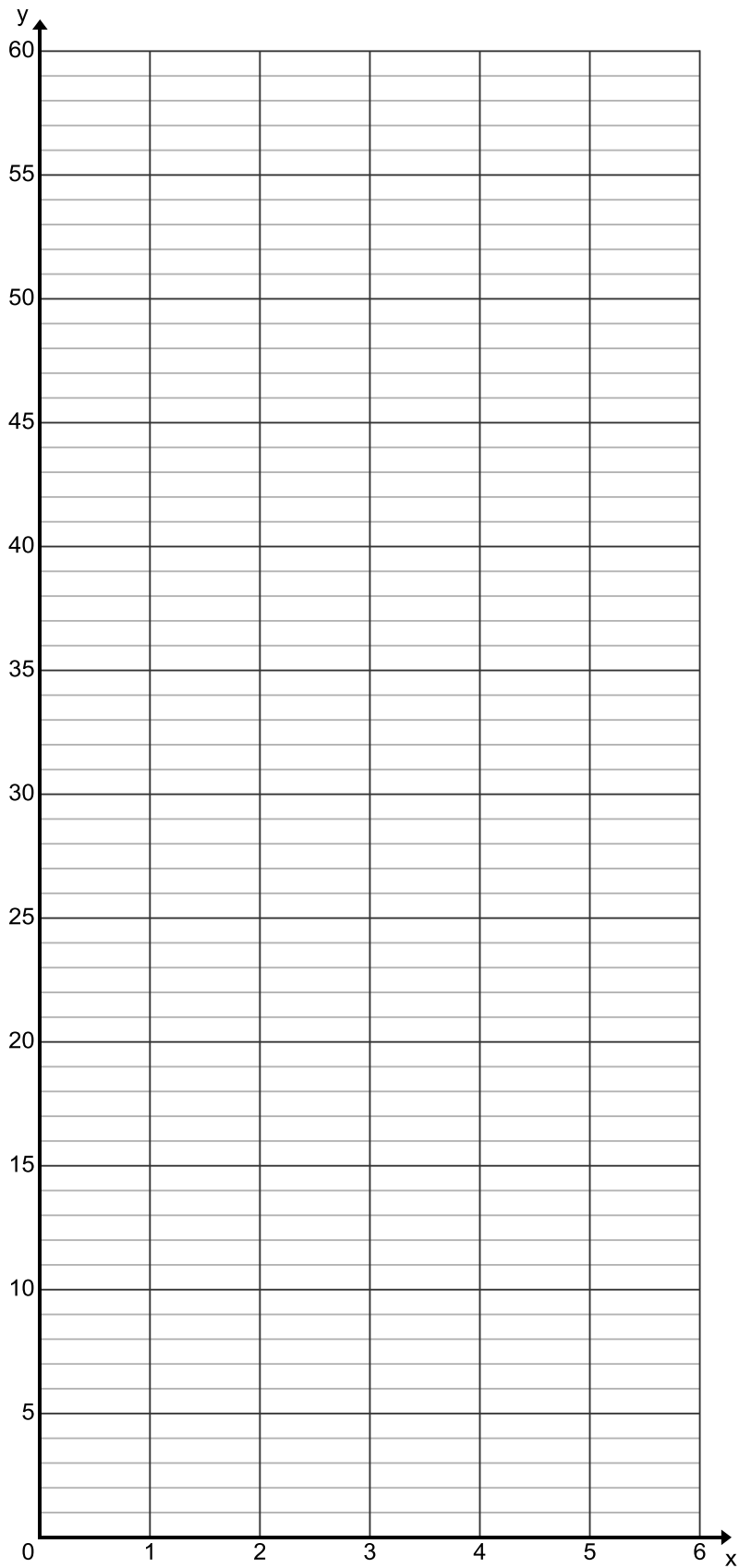


Start by counting the straws and papers for a cube with edges of 1 foot, then count for the 2nd and 3rd cube.

Next, use the patterns or your own drawings to fill in the rows for 0, 4 and 5.

x (in feet)	How many straws Jason would need?	How many sheets of construction paper Jason would need to cover the sides?	What is the volume of the cube that Jason makes?
0			
1			
2			
3			
4			
5			

← Note: the volume of a cube is the number of 1 ft cubes that will fit inside it.



Use the data from your table and three different colors to make three graphs in this coordinate plane.

One graph will be for the number of straws.

A second will be for the number of 1ft by 1ft squares of paper.

The third will be for the volume.

1. How many straws do you think Jason will need if $x = 6$?
 - a. How do you know? Explain the pattern.
 - b. Is there an equation you can create to represent the number of straws Jason will need given any x value? Write the equation.
 - c. What does this graph look like? What type of function is this?
 - d. How many straws will Jason need if $x = 10$? Show your work.

2. How many sheets of construction paper do you think Jason will need if $x = 6$?
 - a. How do you know? Explain the pattern.
 - b. Is there an equation you can create to represent the sheets of paper Jason will need given any x value? Write the equation.
 - c. What does this graph look like? What type of function is this?
 - d. How many sheets of paper will Jason need if $x = 10$? Show your work.

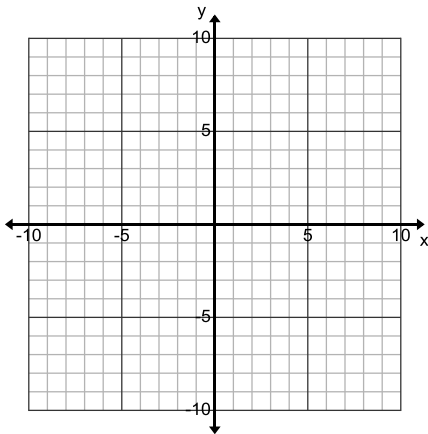
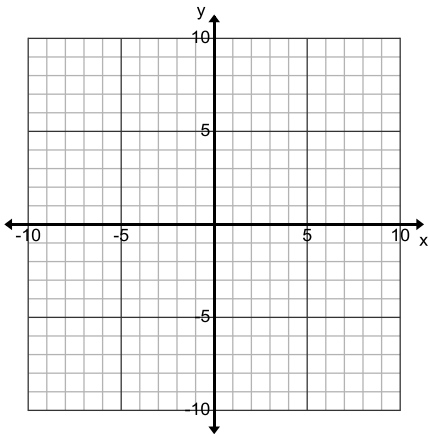
3. What would be the volume if Jason builds a cube with $x = 6$?
 - a. How do you know? Explain the pattern.
 - b. Is there an equation you can create to represent the volume for any x value? Write the equation.
 - c. What does this graph look like? What type of function is this?
 - d. What will be the volume if $x = 10$? Show your work.

This page was intentionally left blank.

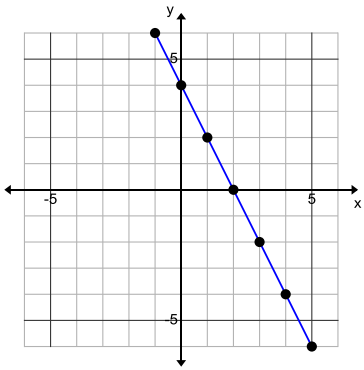
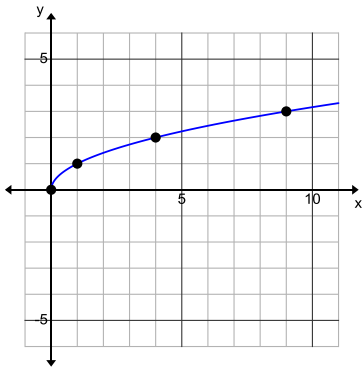
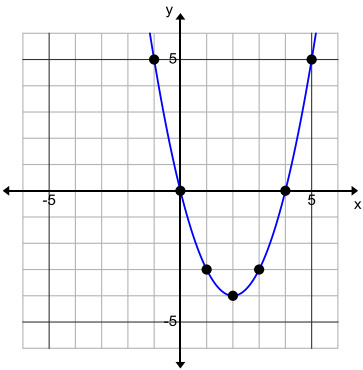
Part 1: For each of the functions given below, find the indicated values.

$f(x) = 3x$ $f(5) =$ $f(-2) =$	$g(x) = x^2$ $g(4) =$ $g(-3) =$	$h(x) = x^3$ $h(3) =$ $h(-5) =$
$f(x) = \sqrt{x}$ $f(16) =$ $f(100) =$	$g(x) = 3^x$ $g(4) =$ $g(-3) =$	$h(x) = x^3 - 3x^2$ $h(4) =$ $h(-4) =$

Part 2: Draw the graphs of the following functions.

 $f(x) = 3x - 4$	 $g(x) = x^2 - 6$
--	--

Part 3: Identify the function in each graph.

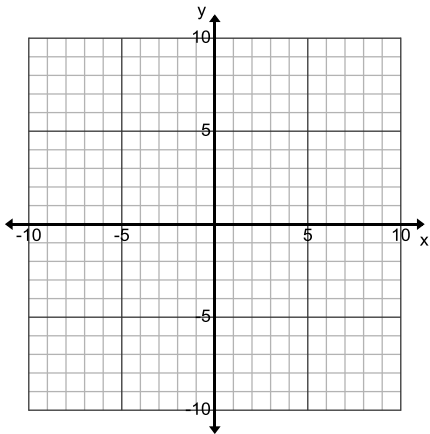
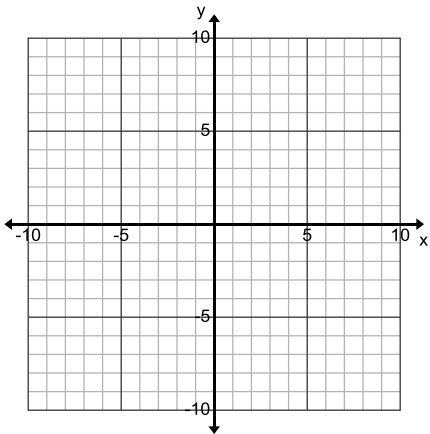
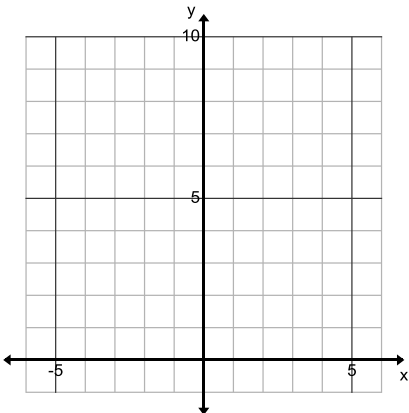
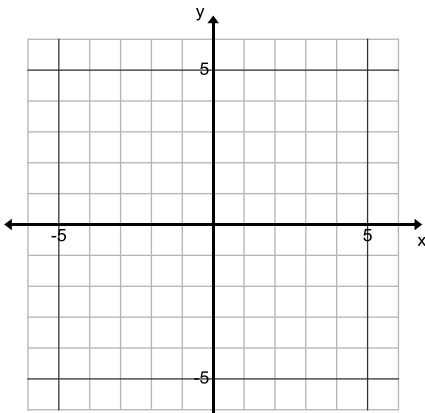
		
$f(x) =$	$g(x) =$	$h(x) =$

Note: $f(x) = 5x$ is read “f of x equals five x” and $f(4)$ is read “f of 4” and is the y-value or function value at $x=4$.

For each of the functions given below, find the indicated values.

$f(x) = -x + 2$	$g(x) = 3x^2$	$h(x) = x^3 - 4$
$f(5) =$	$g(4) =$	$h(3) =$
$f(-2) =$	$g(-3) =$	$h(-5) =$
$f(x) = \sqrt{x}$	$g(x) = 4^x$	$h(x) = x^2 + 4x + 3$
$f(9) =$	$g(2) =$	$h(2) =$
$f(81) =$	$g(-3) =$	$h(-3) =$

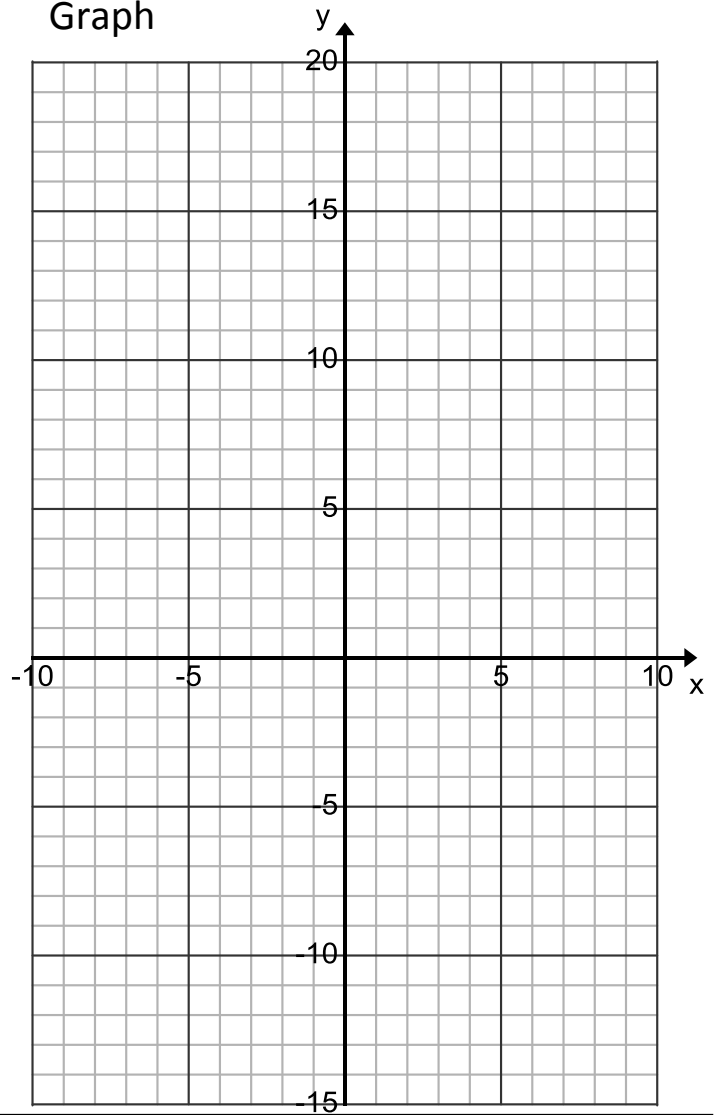
Draw the graphs of the following functions by calculating function values for each input.

	$f(x) = \frac{1}{2}x - 2$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>-2</td><td></td></tr> <tr><td>0</td><td></td></tr> <tr><td>2</td><td></td></tr> <tr><td>4</td><td></td></tr> <tr><td>6</td><td></td></tr> </tbody> </table>	x	y	-2		0		2		4		6			$g(x) = x^2 - 4$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>-2</td><td></td></tr> <tr><td>-1</td><td></td></tr> <tr><td>0</td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td>2</td><td></td></tr> </tbody> </table>	x	y	-2		-1		0		1		2	
x	y																										
-2																											
0																											
2																											
4																											
6																											
x	y																										
-2																											
-1																											
0																											
1																											
2																											
	$k(x) = 3^x$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>-2</td><td></td></tr> <tr><td>-1</td><td></td></tr> <tr><td>0</td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td>2</td><td></td></tr> </tbody> </table>	x	y	-2		-1		0		1		2			$h(x) = \sqrt{x + 5}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>-5</td><td></td></tr> <tr><td>-4</td><td></td></tr> <tr><td>-1</td><td></td></tr> <tr><td>4</td><td></td></tr> <tr><td>11</td><td></td></tr> </tbody> </table>	x	y	-5		-4		-1		4		11	
x	y																										
-2																											
-1																											
0																											
1																											
2																											
x	y																										
-5																											
-4																											
-1																											
4																											
11																											

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = 2^x$$

Situation

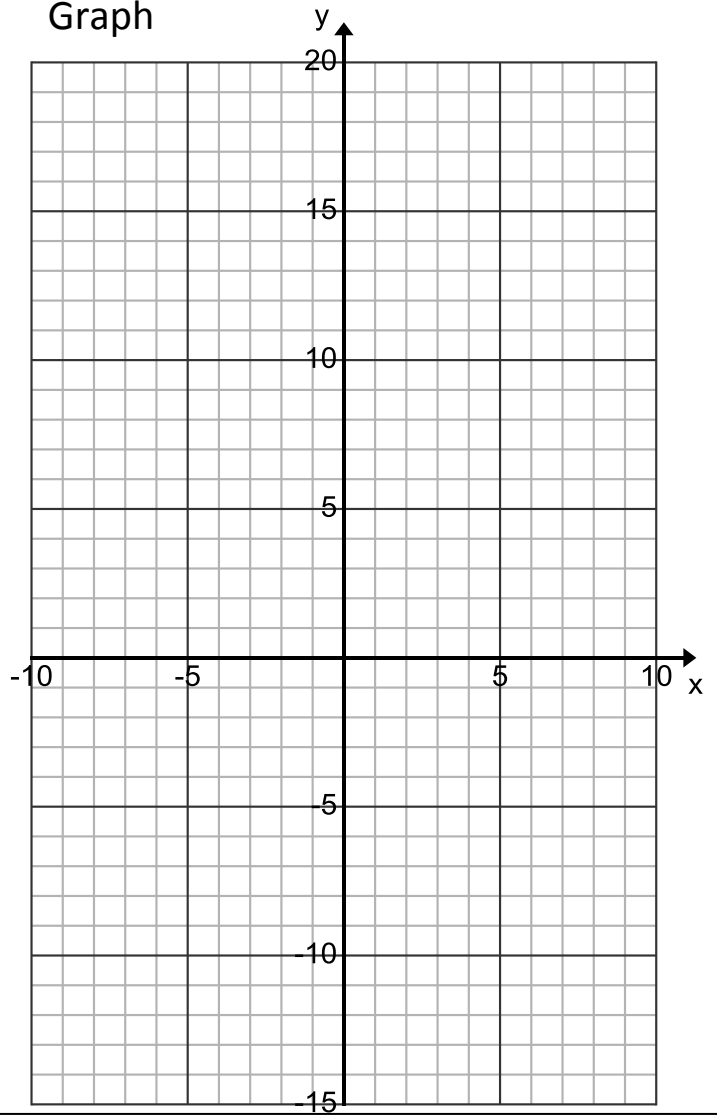
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512

Graph



Model Equation

$$f(x) =$$

Situation

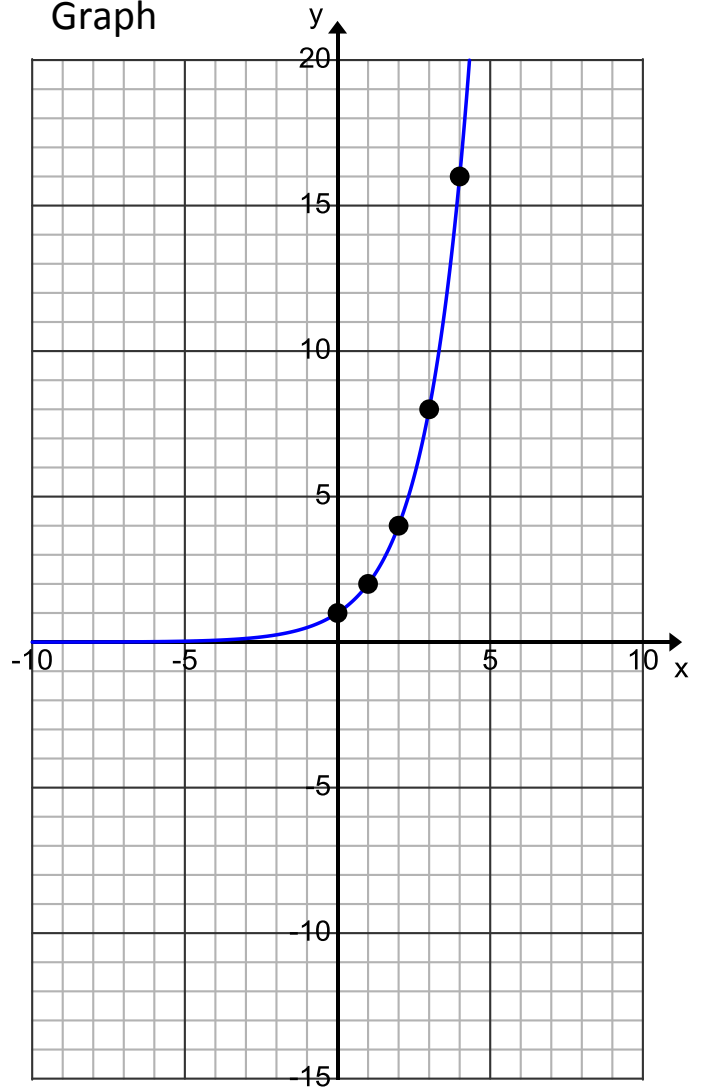
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

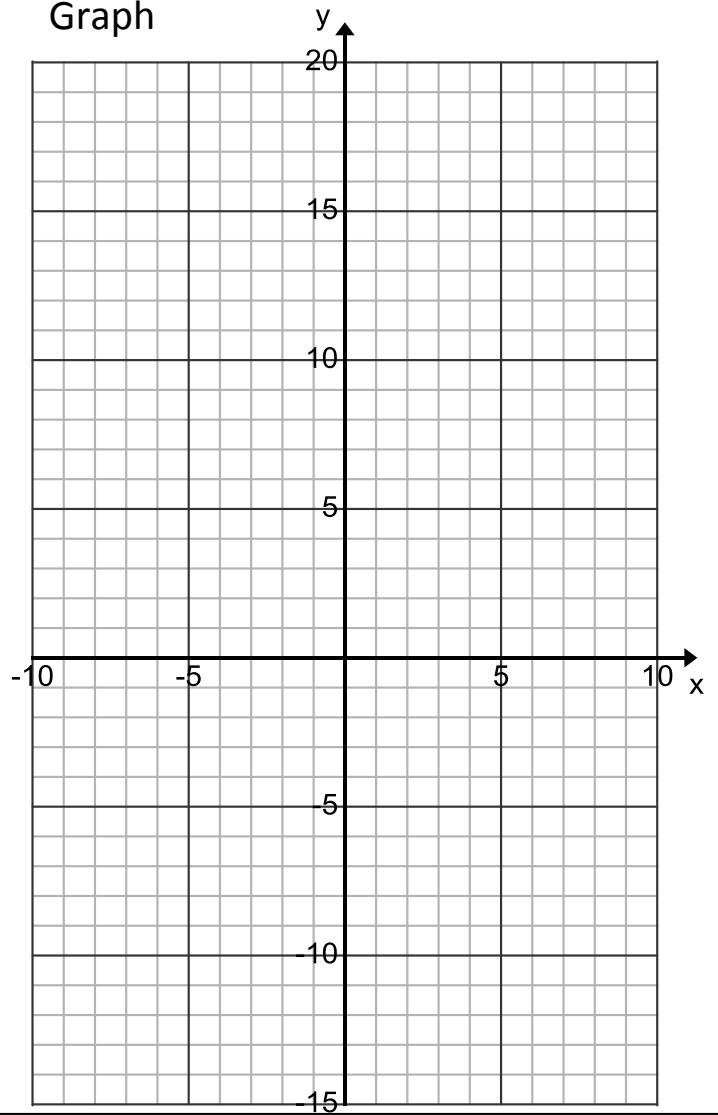
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

Ezra starts with just one block, but doubles his amount on the first day. He continues to double his amount every day.

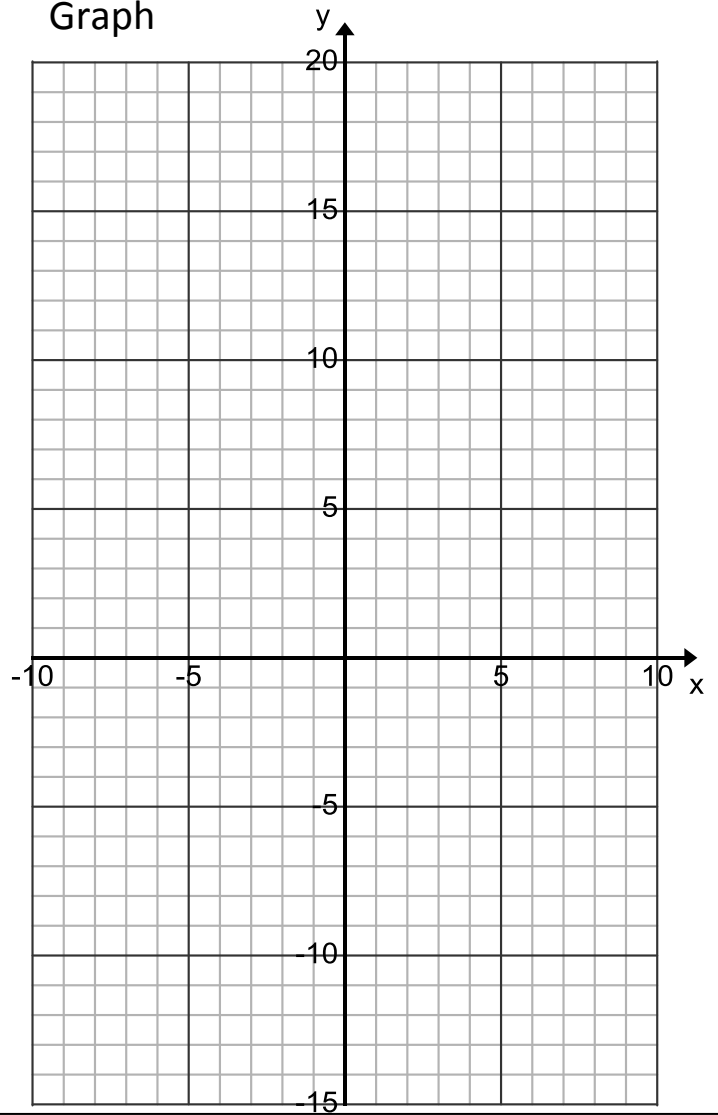
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

Drawings of the first five days.

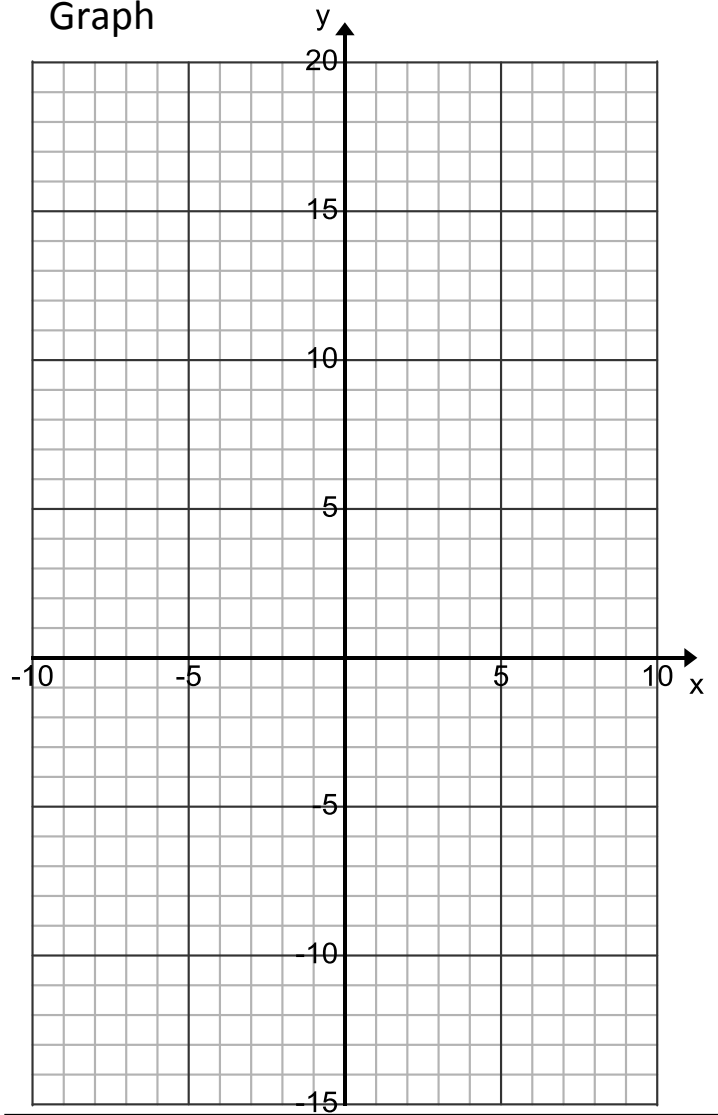


Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = \sqrt{x}$$

Situation

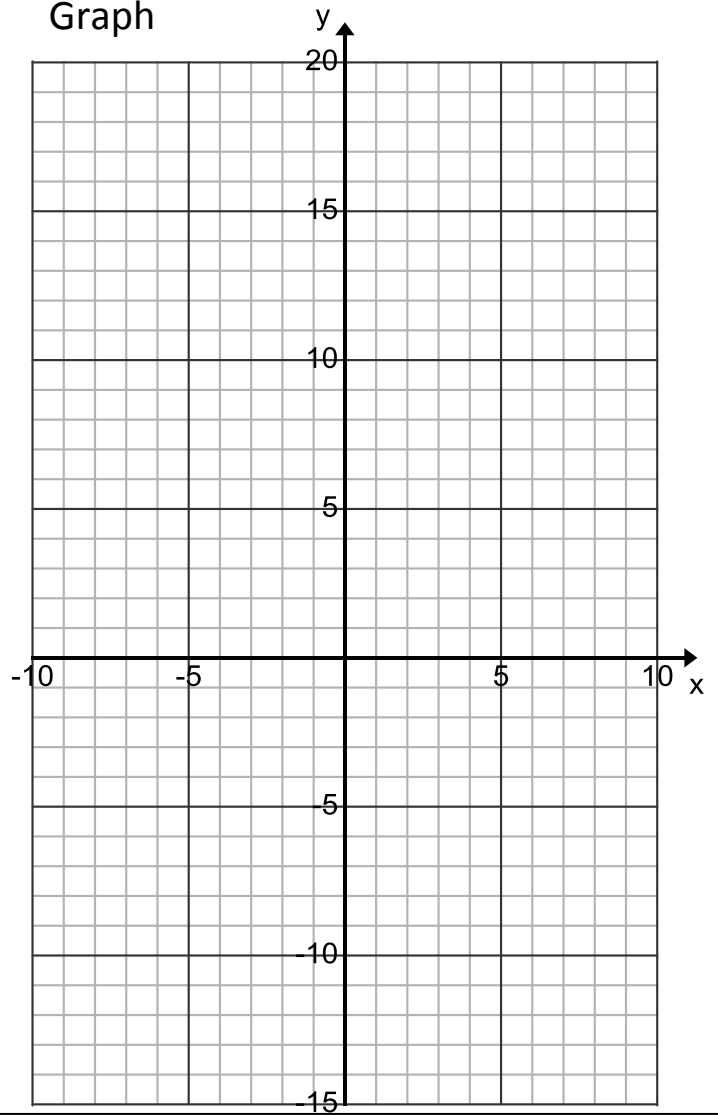
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	0
1	1
2	1.41
3	1.73
4	2
5	2.24
6	2.45
7	2.65
8	2.83
9	3

Graph



Model Equation

$$f(x) =$$

Situation

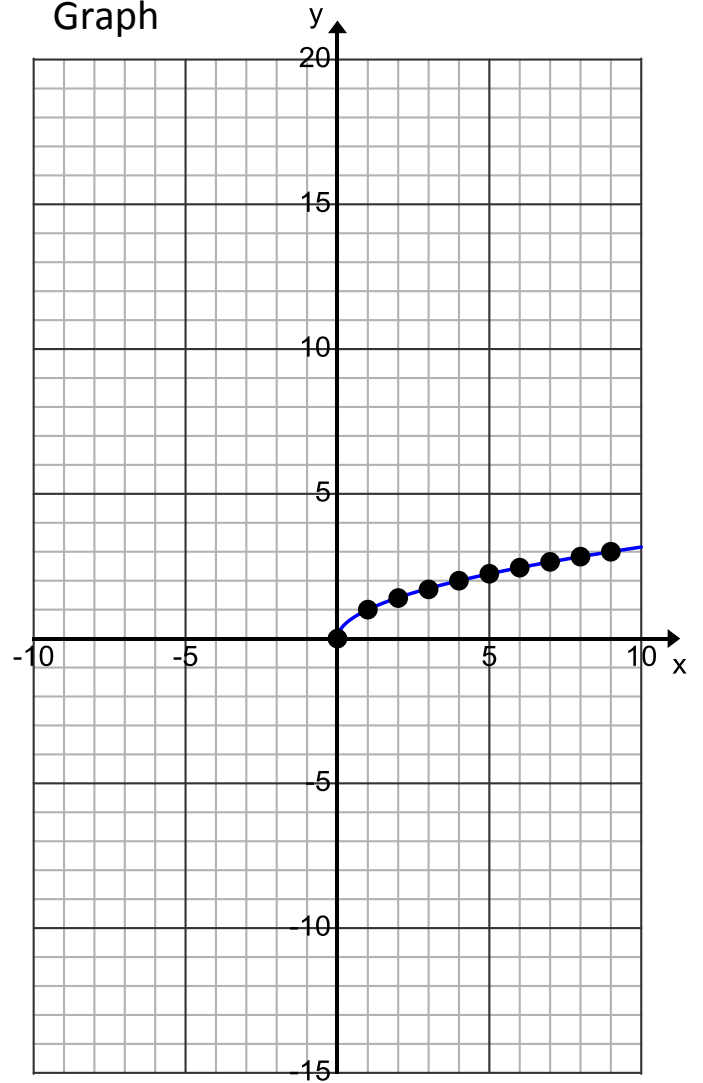
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

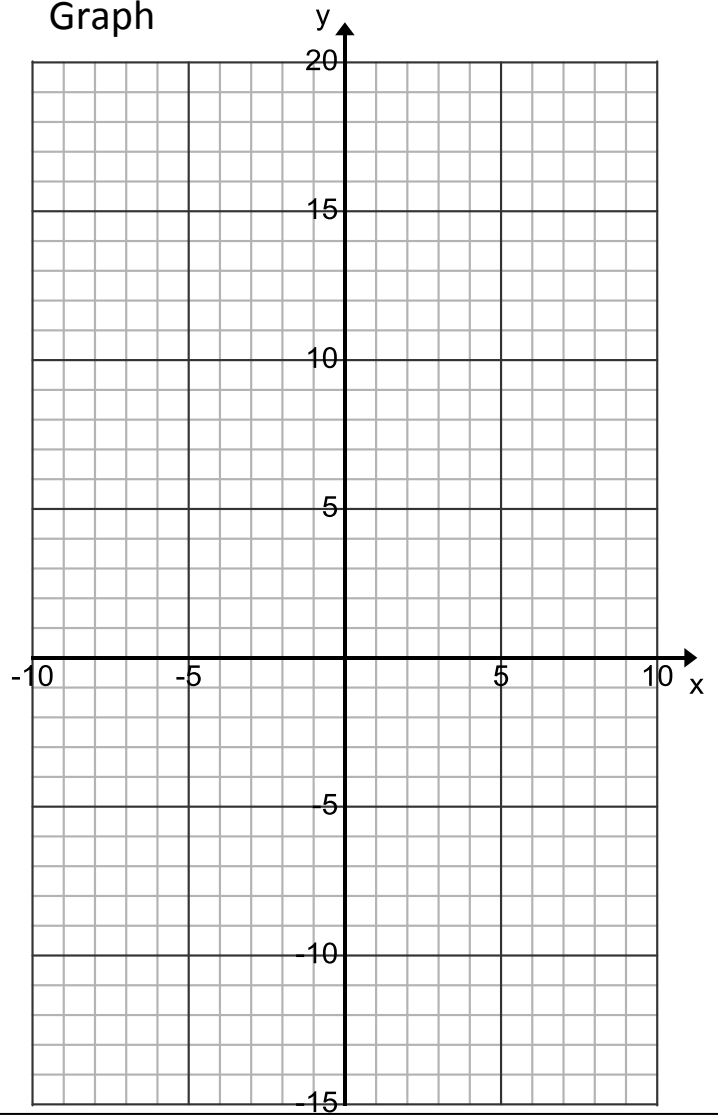
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

Simon loves squares. He doesn't have one on the first day, but every day after he makes a square with an area that is one larger than the day before. Then he measures the side of the square.

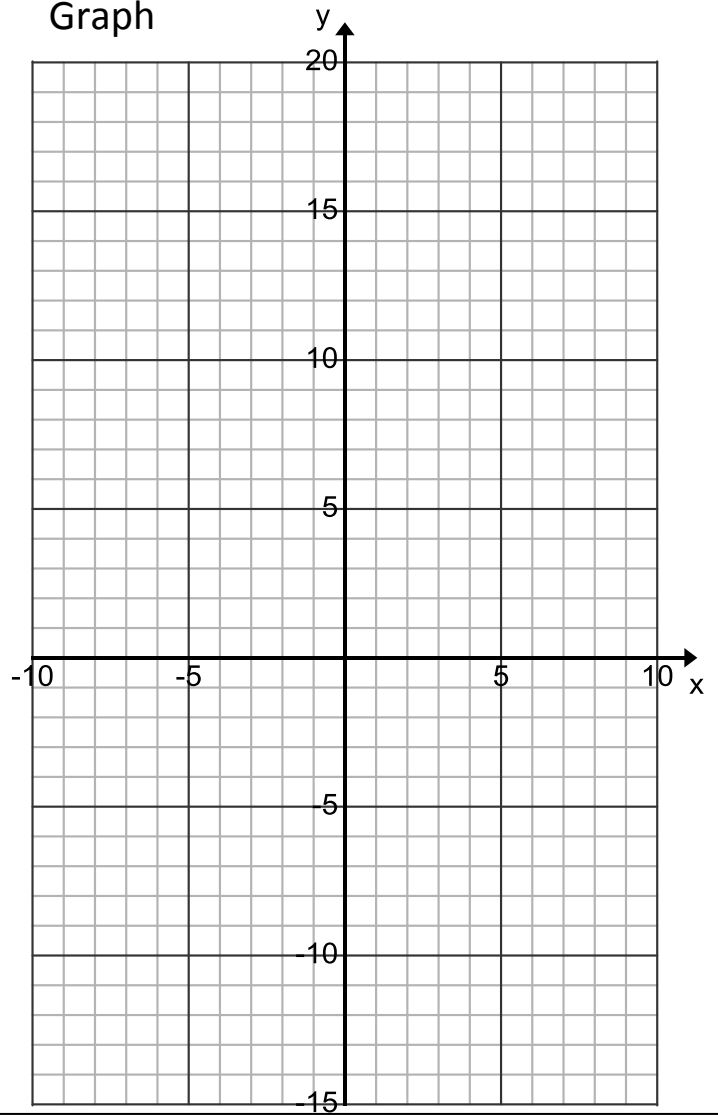
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

Drawing

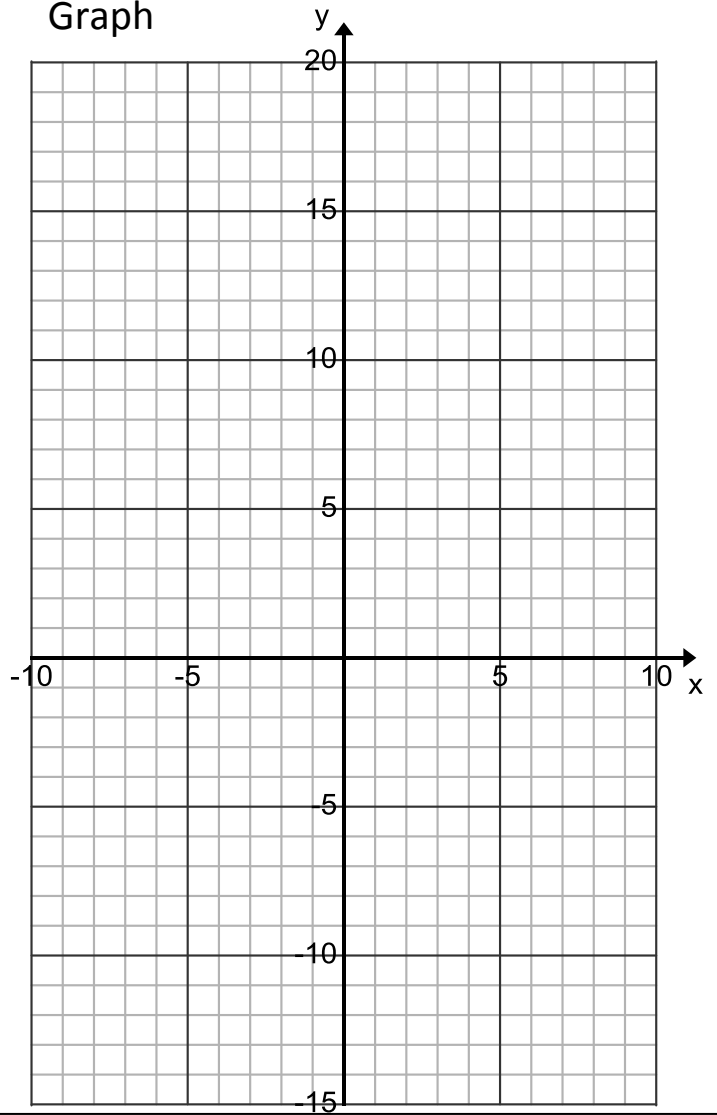
? ?
1 ?
2 ?
3 ?
4

Record your observations and questions here:

Data Table

x	f(x)
0	0
1	3
2	8
3	15
4	24
5	35
6	48
7	63
8	80
9	99

Graph



Model Equation

$$f(x) =$$

Situation

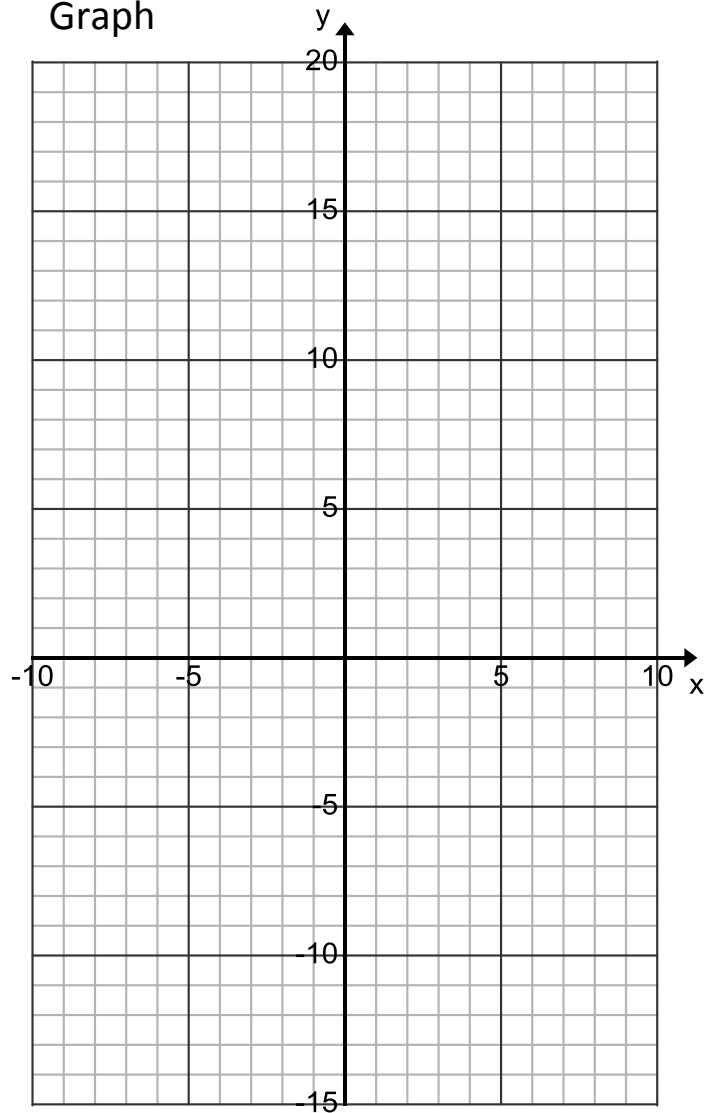
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = x^2 + 2x$$

Situation

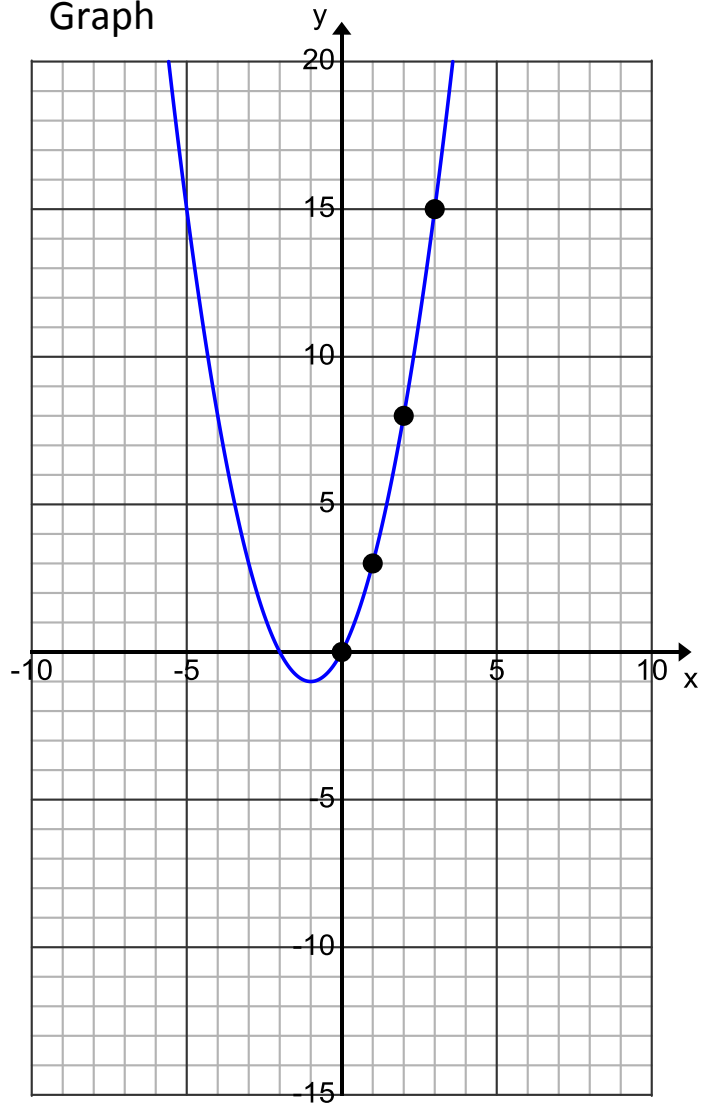
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

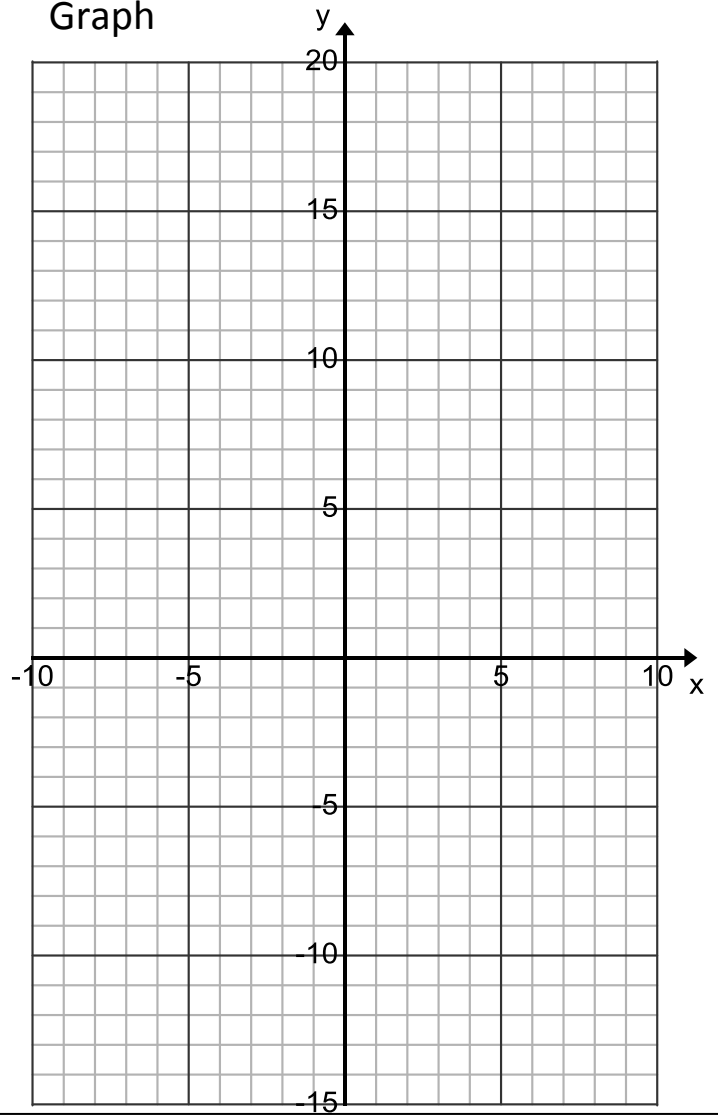
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph

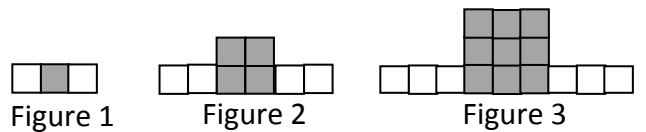


Model Equation

$$f(x) =$$

Situation

Drawing

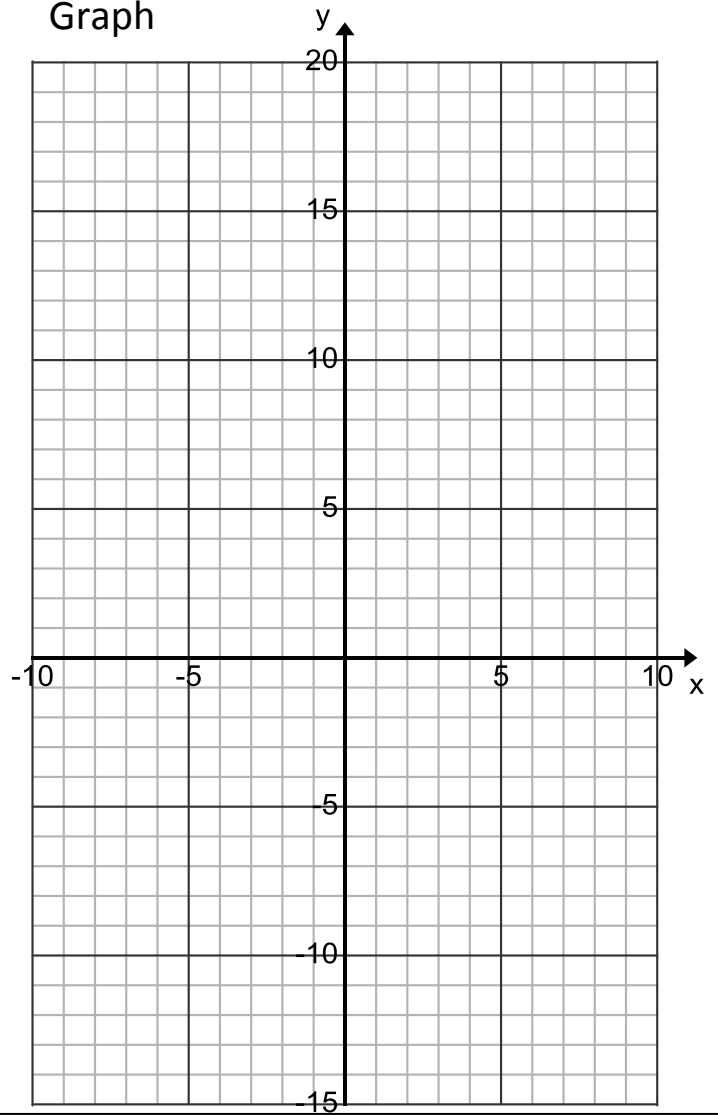


Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

Bella is building a pattern with square tiles that she calls "Squares with Wings." Each square has two straight wings that are as long as the side of the square.

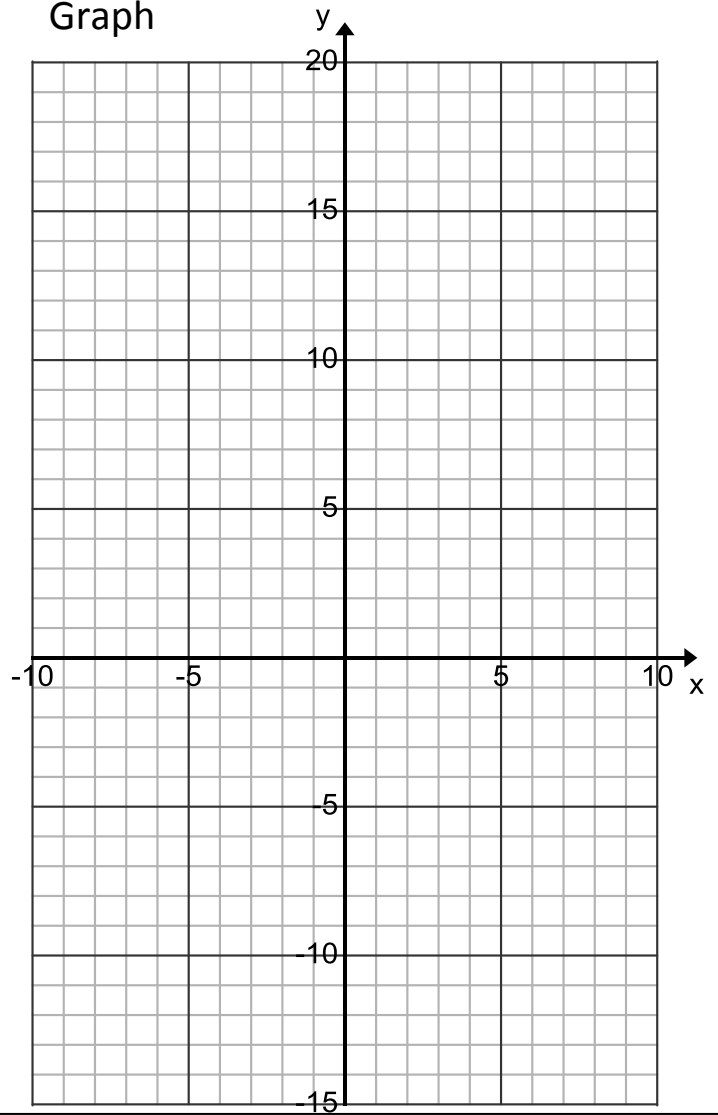
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = x^3$$

Situation

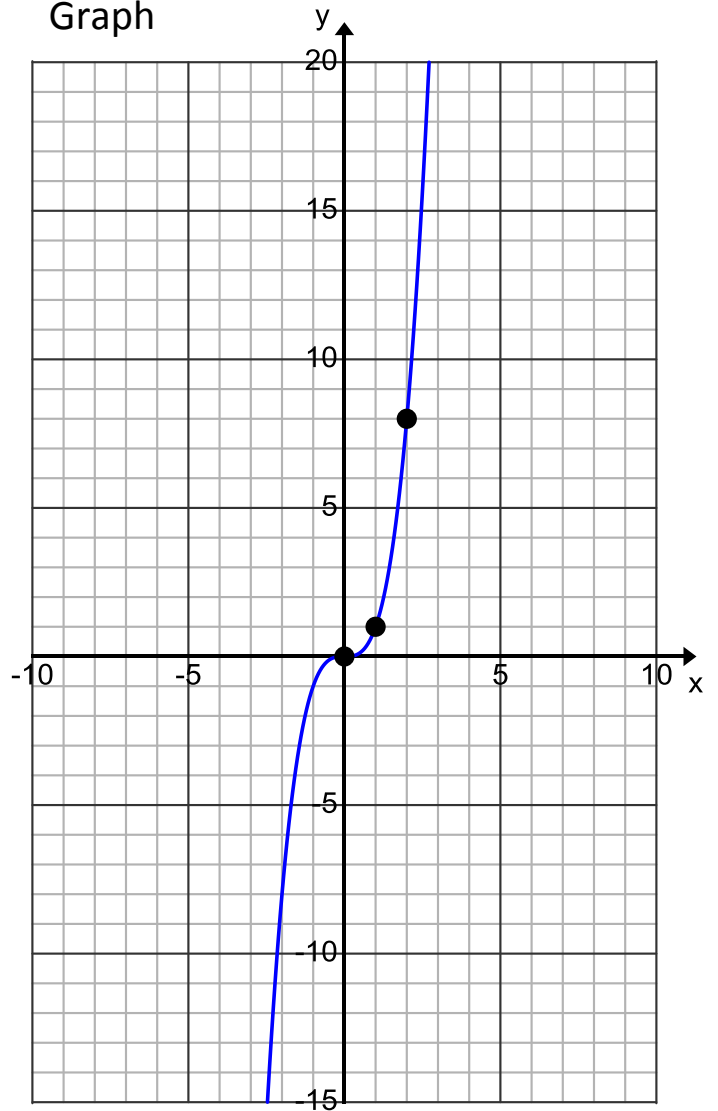
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

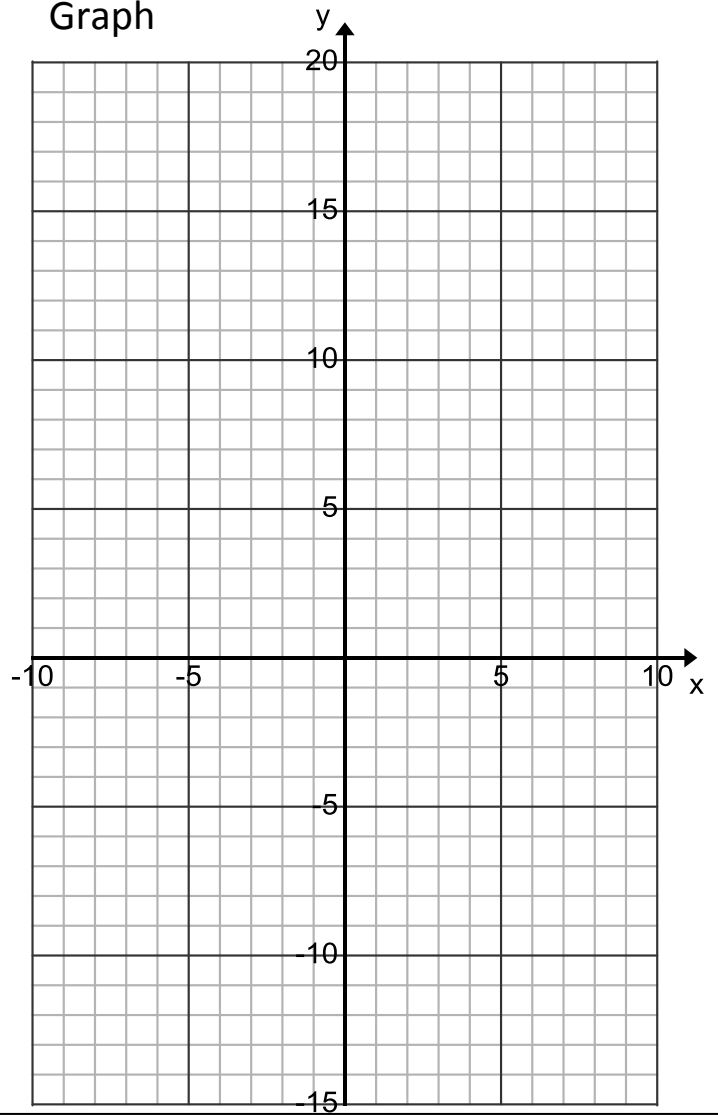
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	0
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729

Graph



Model Equation

$$f(x) =$$

Situation

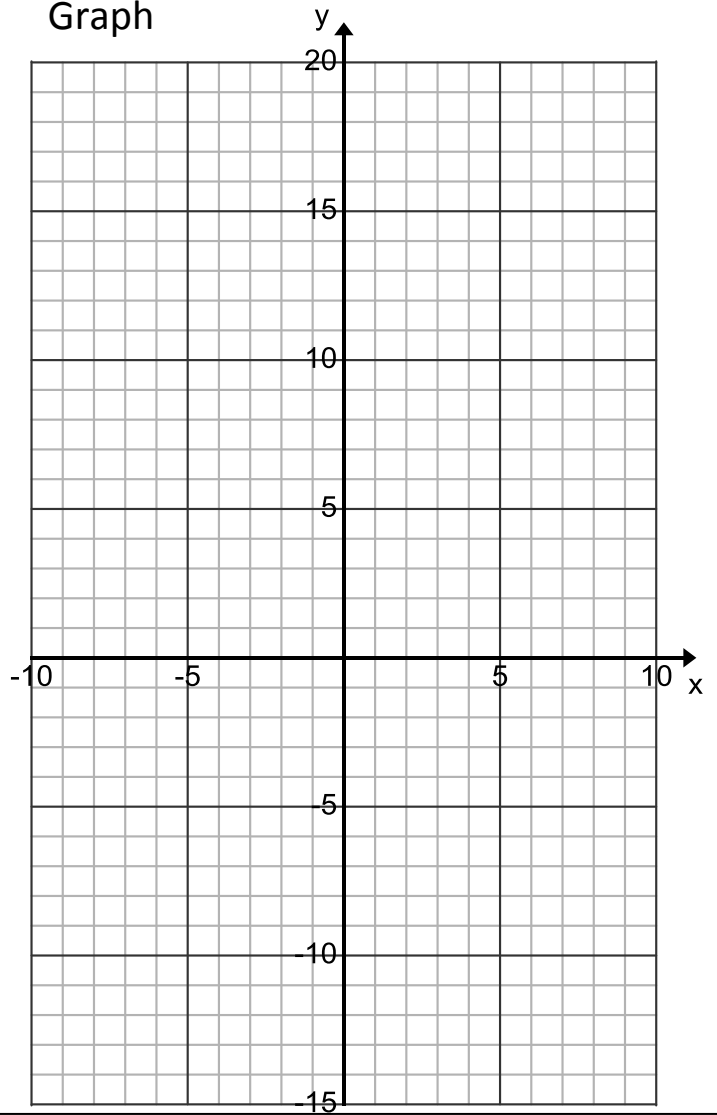
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

How many unit cubes does it take to build a larger cube that is "x" long on each edge?

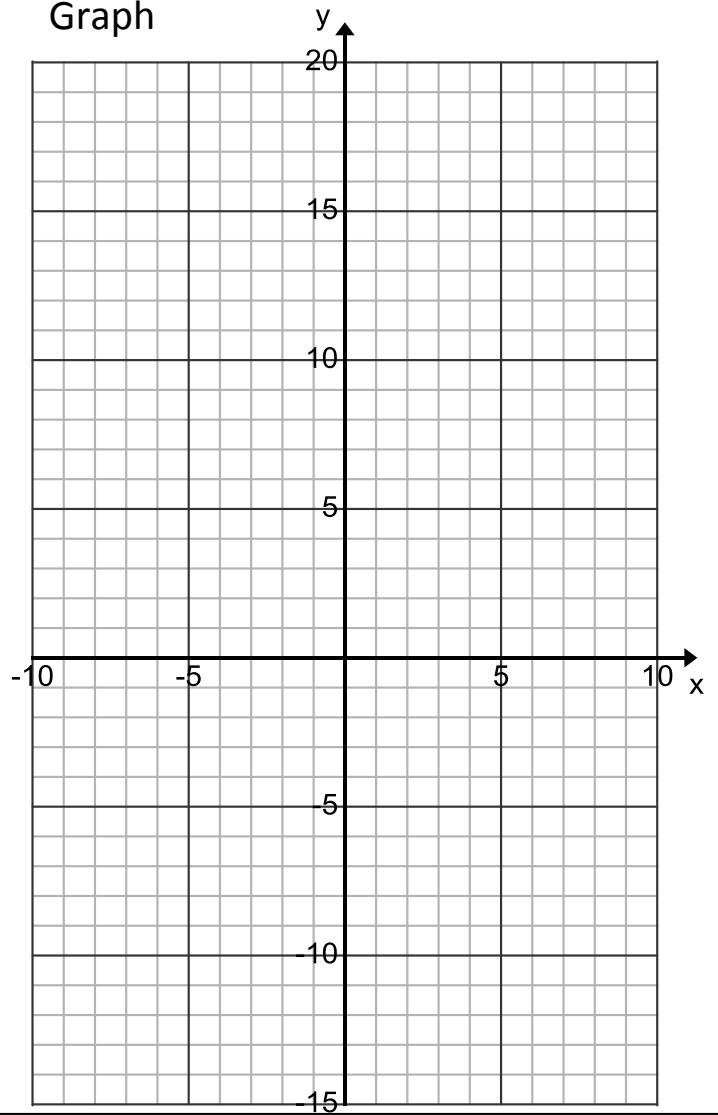
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph

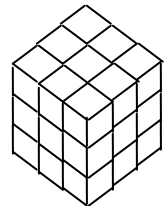
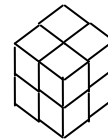


Model Equation

$$f(x) =$$

Situation

Drawing

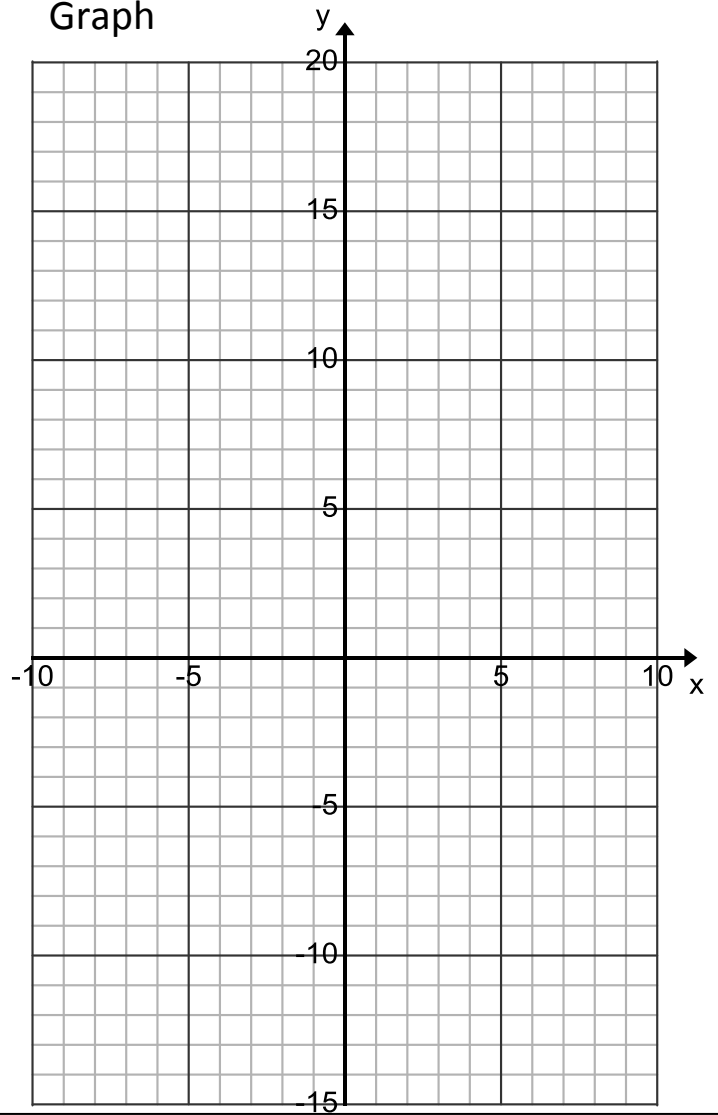


Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) = 3x + 1$$

Situation

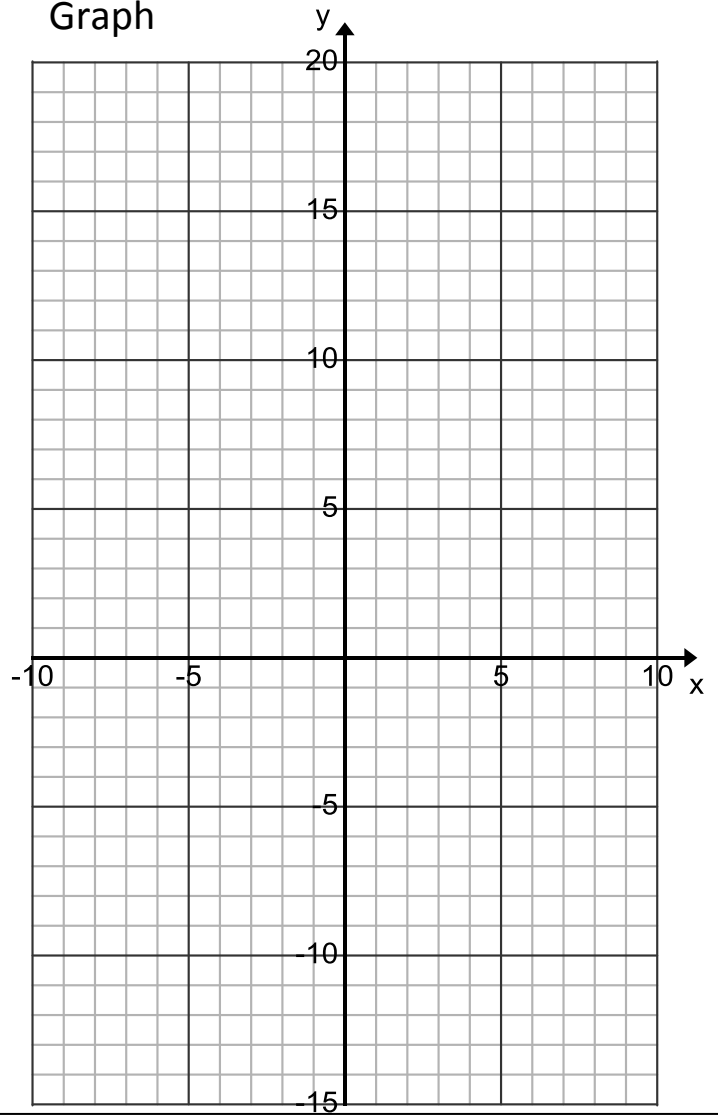
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	1
1	4
2	7
3	10
4	13
5	16
6	19
7	22
8	25
9	28

Graph



Model Equation

$$f(x) =$$

Situation

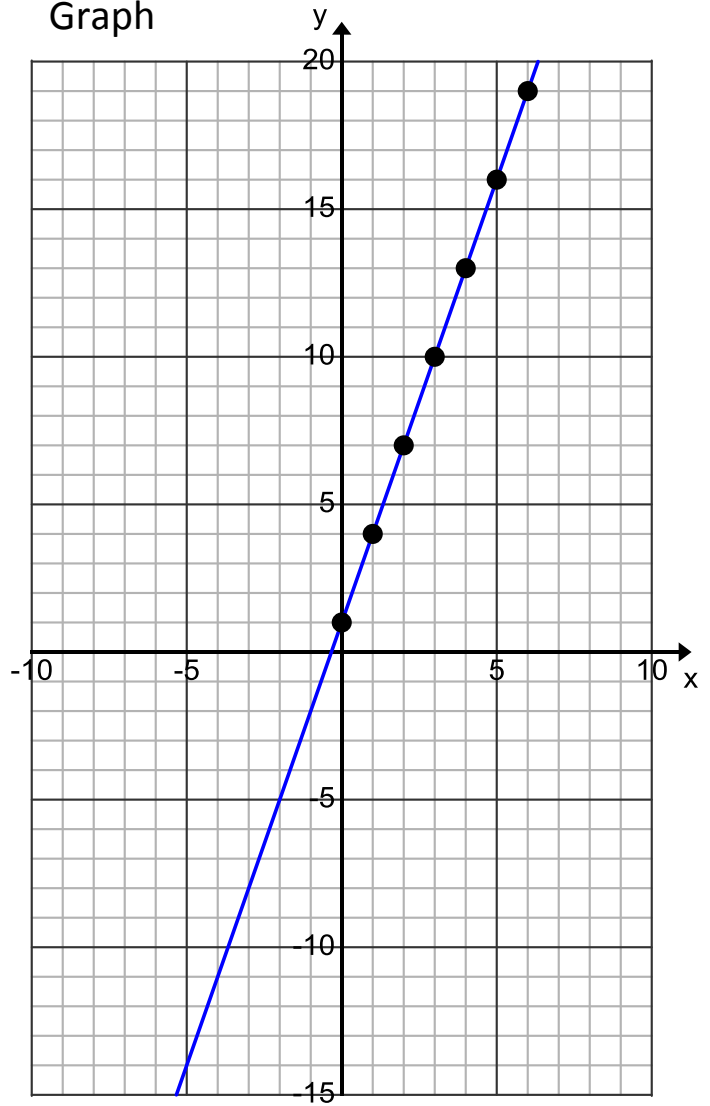
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

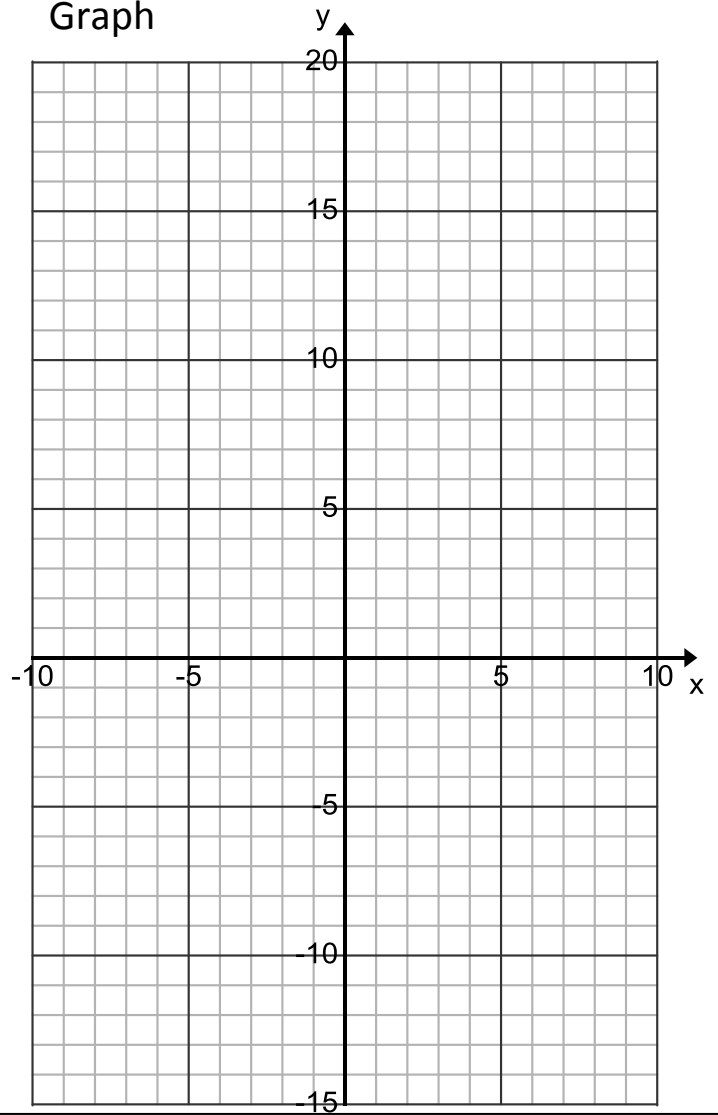
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

Angel is saving money for a new skateboard. He has \$1 and knows if he stops buying cookies at lunch he can save \$3 each week. How much will he have in x weeks?

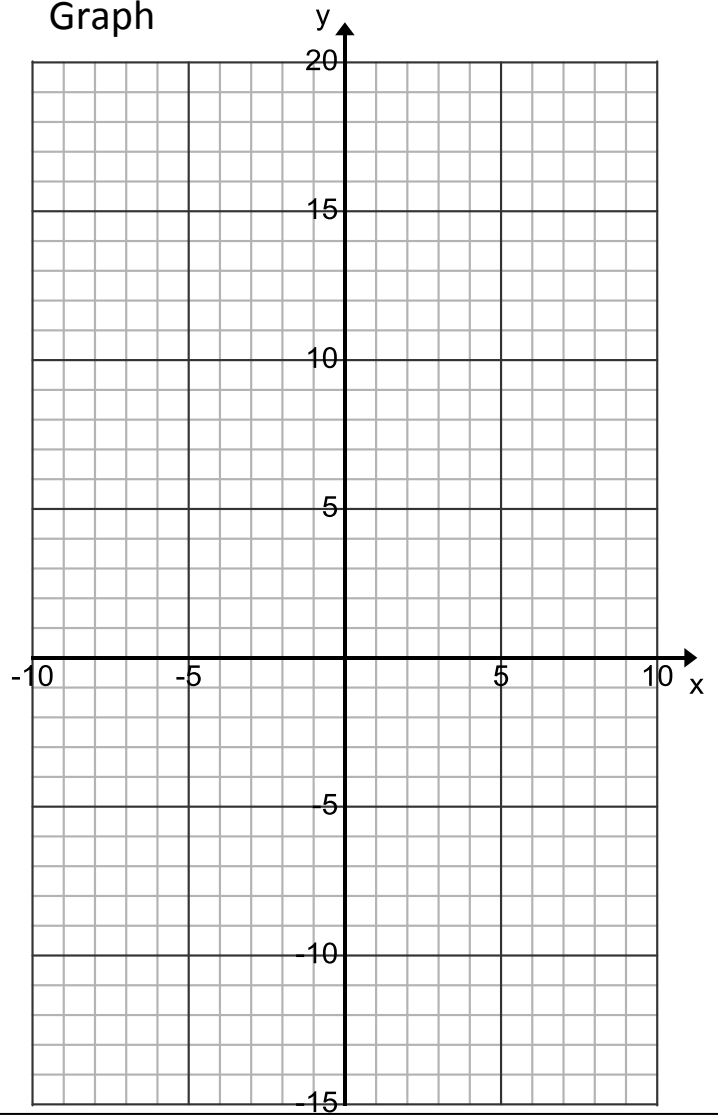
Drawing

Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

$$f(x) =$$

Situation

Drawing

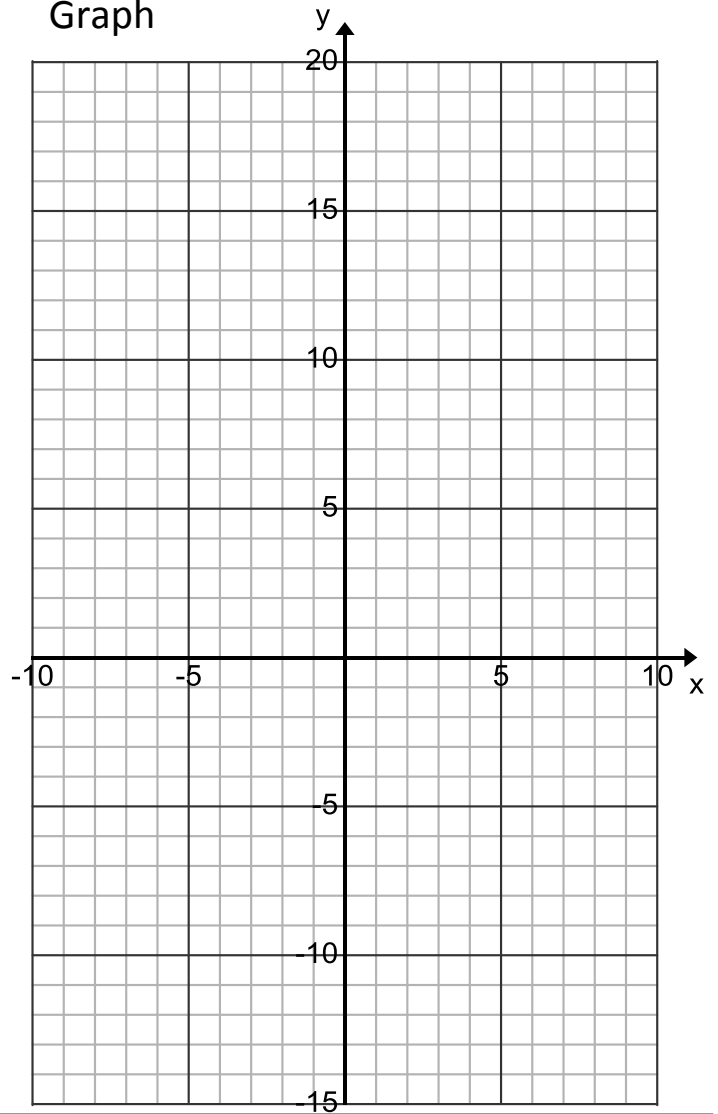


Record your observations and questions here:

Data Table

x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Graph



Model Equation

Situation

Drawing

Record your observations and questions here:

Summary of Key Features from Each of the Five Representations

Directions: In each of the boxes below, write several sentences explaining how all of your pages match. Each box is about a different key feature of the function. In your writing, include specific facts from your different representations of the function. Be sure to include observations from all five representations: table, graph, equation, drawing and story.

Y-intercept: How does the y-intercept appear in each representation? What does it mean?

Other Points: Explain how pairs of inputs and outputs are the same in each representation. Explain what a few of them mean.

Rate of Change: How is the function growing? How do you see this growth in each representation? Explain how the rate of change matches across all of the representations.

This page was intentionally left blank.

Domain of a function

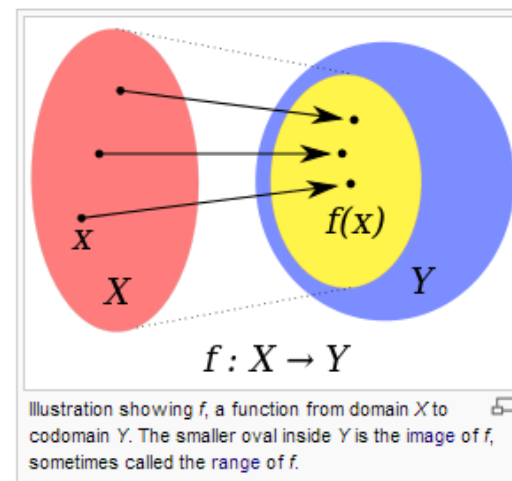
From Wikipedia, the free encyclopedia

In **mathematics**, the **domain of definition** or simply the **domain** of a **function** is the set of "input" or **argument** values for which the function is defined. That is, the function provides an "output" or **value** for each member of the domain.^[1] The set of values the function may take is termed the **range** of the function.

For instance, the domain of **cosine** is the set of all **real numbers**, while the domain of the **square root** consists only of numbers greater than or equal to 0 (ignoring **complex numbers** in both cases). For a function whose domain is a subset of the **real numbers**, when the function is represented in an *xy* **Cartesian coordinate system**, the domain is represented on the *x*-axis.

Contents [hide]

- 1 Formal definition
- 2 Natural domain
- 3 Domain of a partial function
- 4 Category theory
- 5 Real and complex analysis
- 6 More examples
- 7 See also
- 8 References



Formal definition

[edit]

Given a **function** $f: X \rightarrow Y$, the set X is the **domain** of f ; the set Y is the **codomain** of f . In the expression $f(x)$, x is the **argument** and $f(x)$ is the **value**. One can think of an argument as an input to the function, and the value as the output.

The **image** (sometimes called the **range**) of f is the set of all values assumed by f for all possible x ; this is the set $\{f(x) \mid x \in X\}$. The image of f can be the same set as the codomain or it can be a proper subset of it. It is in general smaller than the codomain; it is the whole codomain if and only if f is a **surjective function**.

A well-defined function must carry every element of its domain to an element of its codomain. For example, the function f defined by

$$f(x) = 1/x$$

has no value for $f(0)$. Thus, the set of all **real numbers**, \mathbb{R} , cannot be its domain. In cases like this, the function is either defined on $\mathbb{R} - \{0\}$ or the "gap is plugged" by explicitly defining $f(0)$. If we extend the definition of f to

$$f(x) = \begin{cases} 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

then f is defined for all real numbers, and its domain is \mathbb{R} .

Any function can be restricted to a **subset** of its domain. The **restriction** of $g: A \rightarrow B$ to S , where $S \subseteq A$, is written $g|_S: S \rightarrow B$.

This page was copied from Wikipedia on 4/16/2013. | http://en.wikipedia.org/wiki/Domain_of_a_function

Range (mathematics)

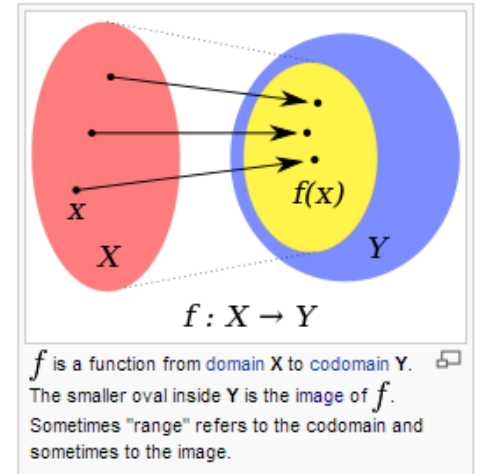
From Wikipedia, the free encyclopedia

This article is about range of a function. For the difference between the largest and smallest numbers in a set, see [range \(statistics\)](#).

In **mathematics**, the **range** of a **function** refers to either the **codomain** or the **image** of the function, depending upon usage. The codomain is a set containing the function's output, whereas the image is only the part of the codomain where the elements are outputs of the function. For example, the function $f(x) = x^2$ is often described as a function from the **real numbers** to the real numbers, meaning that the codomain is \mathbb{R} , but its image is the set of non-negative real numbers. Some books say that range of this function is its codomain, the set of all real numbers, reflecting that the function is real-valued. These books call the actual output of the function the image. This is the current usage for **range in computer science**. Other books say that the range is the function's image, the set of non-negative real numbers, reflecting that a number can be the output of this function if and only if it is a non-negative real number. In this case, the larger set containing the range is called the codomain.^[1] This usage is more common in modern mathematics.

Contents [hide]

- 1 Examples
- 2 Formal definition
- 3 See also
- 4 References



Examples

[edit]

Let f be a function on the **real numbers** $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x$. This function takes as input any real number and outputs a real number two times the input. In this case, the codomain and the image are the same (*i.e.*, the function is a **surjection**), so the range is unambiguous; it is the set of all real numbers.

In contrast, consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sin(x)$. If the word "range" is used in the first sense given above, we would say the range of f is the codomain, all real numbers; but since the output of the **sine** function is always between -1 and 1, "range" in the second sense would say the range is the image, the closed interval from -1 to 1.

Formal definition

[edit]

Standard mathematical notation allows a formal definition of range.

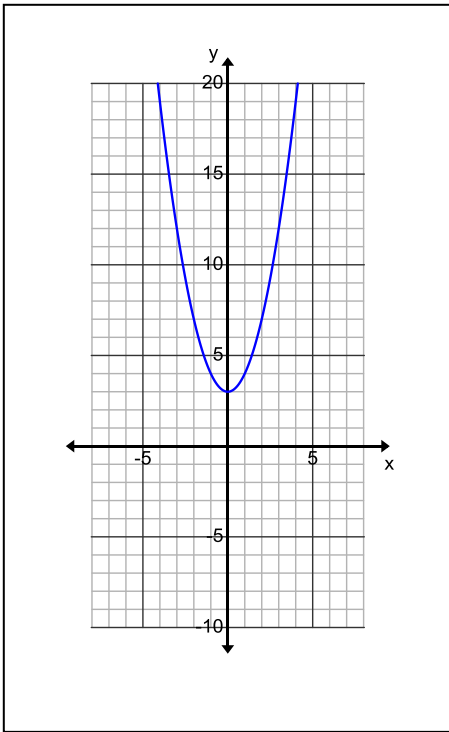
In the first sense, the range of a function must be specified; it is often assumed to be the set of all real numbers, and $\{y \mid \text{there exists an } x \text{ in the domain of } f \text{ such that } y = f(x)\}$ is called the image of f .

In the second sense, the range of a function f is $\{y \mid \text{there exists an } x \text{ in the domain of } f \text{ such that } y = f(x)\}$. In this case, the codomain of f must be specified, but is often assumed to be the set of all real numbers.

In both cases, $\text{image } f \subseteq \text{range } f \subseteq \text{codomain } f$, with at least one of the containments being equality.

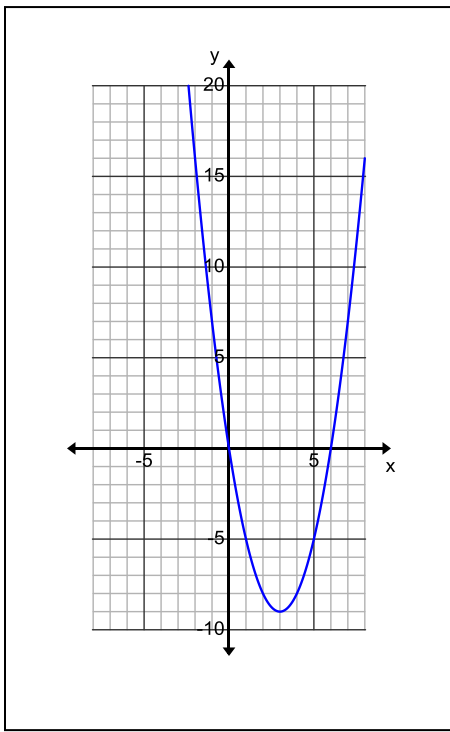
This page was copied from Wikipedia on 4/16/2013. | [http://en.wikipedia.org/wiki/Range_\(mathematics\)](http://en.wikipedia.org/wiki/Range_(mathematics))

Three Quadratic Functions



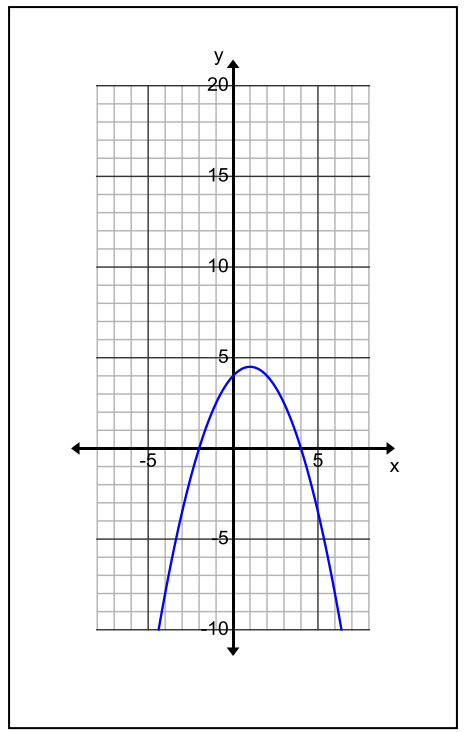
$$f(x) = x^2 + 3$$

Notes/Observations about the domain and range of $f(x)$:



$$g(x) = x^2 - 6x$$

Notes/Observations about the domain and range of $g(x)$:

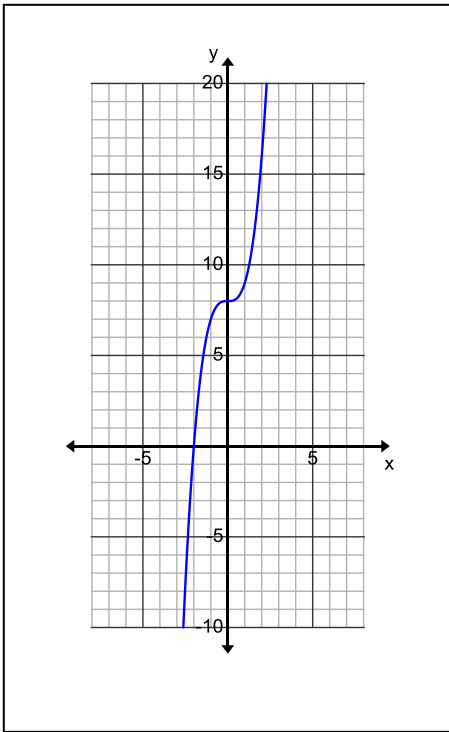


$$h(x) = -\frac{1}{2}x^2 + x + 4$$

Notes/Observations about the domain and range of $h(x)$:

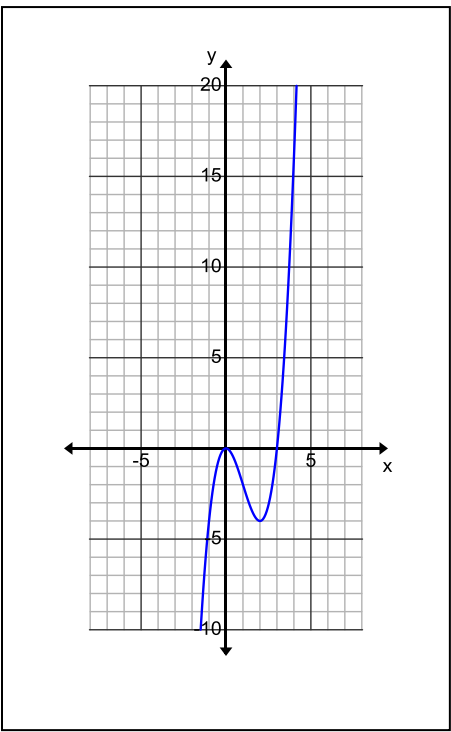
Observations/Conjectures/Conclusions about the domains and ranges of quadratic functions: (Please be sure to explain and justify your statements.)

Three Cubic Functions



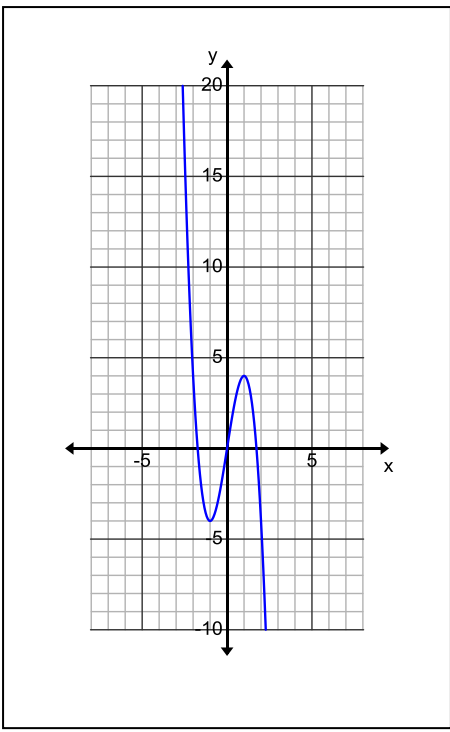
$$f(x) = x^3 + 8$$

Notes/Observations about the domain and range of $f(x)$:



$$g(x) = x^3 - 3x^2$$

Notes/Observations about the domain and range of $g(x)$:

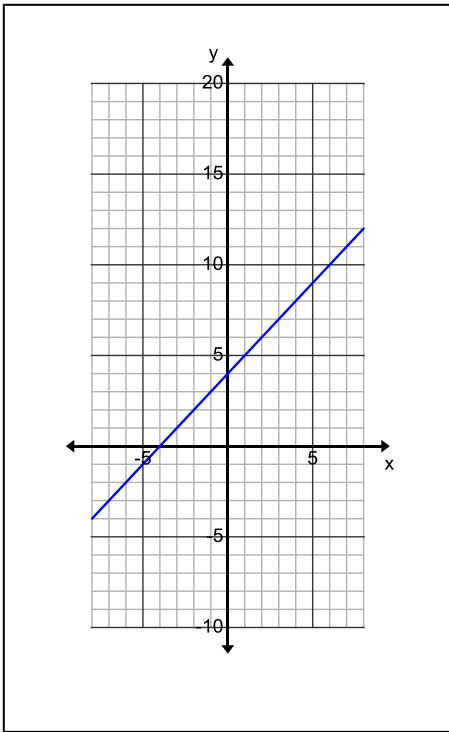


$$h(x) = -2x^3 + 6x$$

Notes/Observations about the domain and range of $h(x)$:

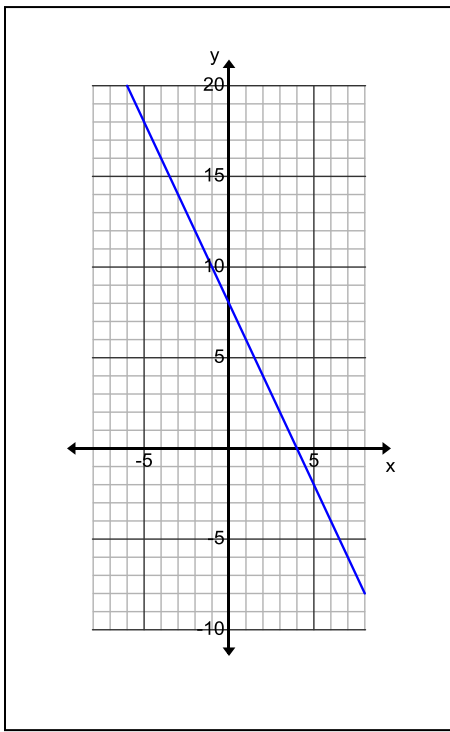
Observations/Conjectures/Conclusions about the domains and ranges of cubic functions: (Please be sure to explain and justify your statements.)

Three Linear Functions



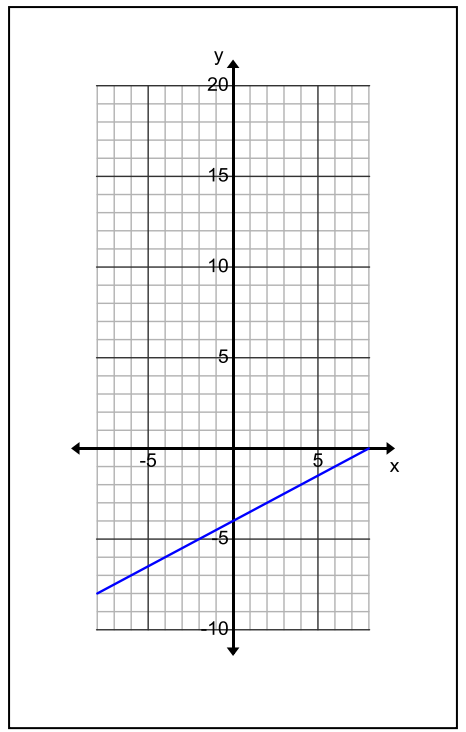
$$f(x) = x + 4$$

Notes/Observations about the domain and range of $f(x)$:



$$g(x) = -2x + 8$$

Notes/Observations about the domain and range of $g(x)$:

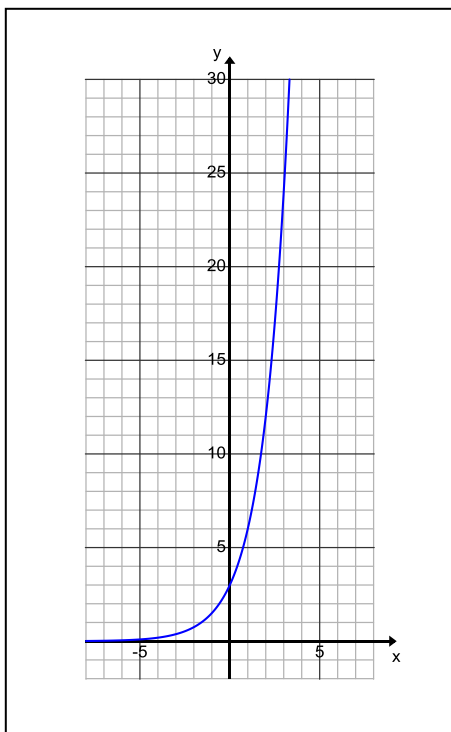


$$h(x) = \frac{1}{2}x - 4$$

Notes/Observations about the domain and range of $h(x)$:

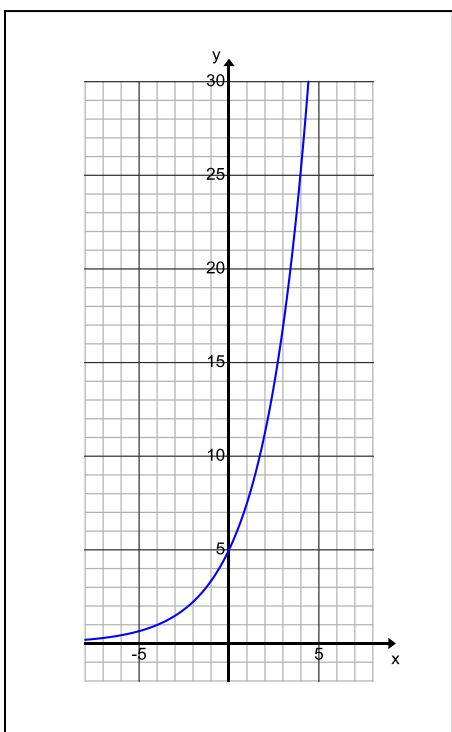
Observations/Conjectures/Conclusions about the domains and ranges of linear functions: (Please be sure to explain and justify your statements.)

Three Exponential Functions



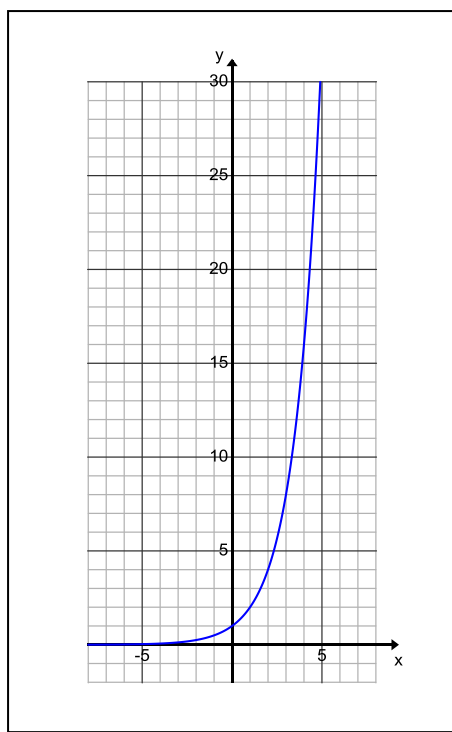
$$f(x) = 3 \cdot 2^x$$

Notes/Observations about the domain and range of $f(x)$:



$$g(x) = 5 \cdot (1.5)^x$$

Notes/Observations about the domain and range of $g(x)$:

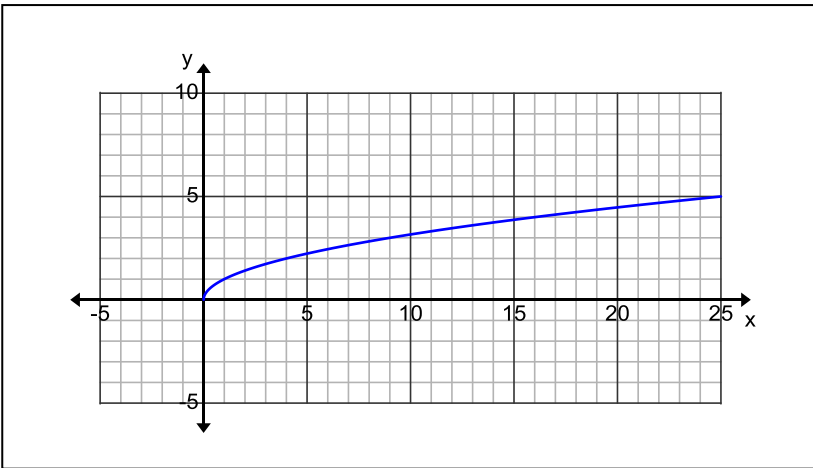


$$h(x) = 2^x$$

Notes/Observations about the domain and range of $h(x)$:

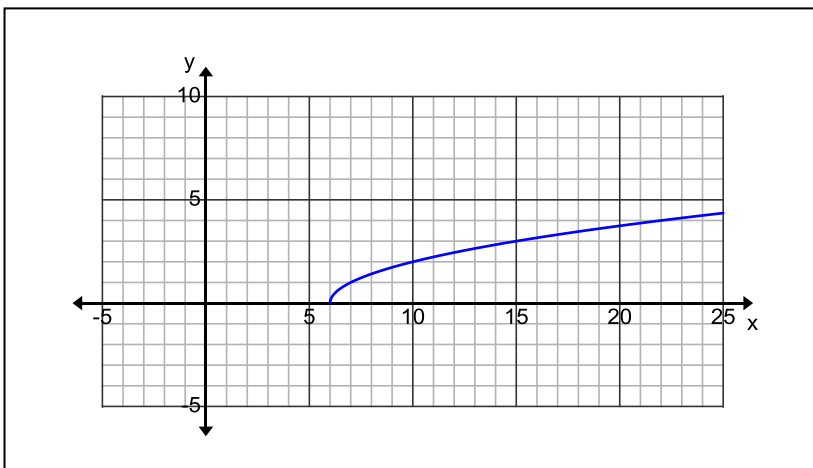
Observations/Conjectures/Conclusions about the domains and ranges of exponential functions:
 (Please be sure to explain and justify your statements.)

Three Radical (Square Root) Functions



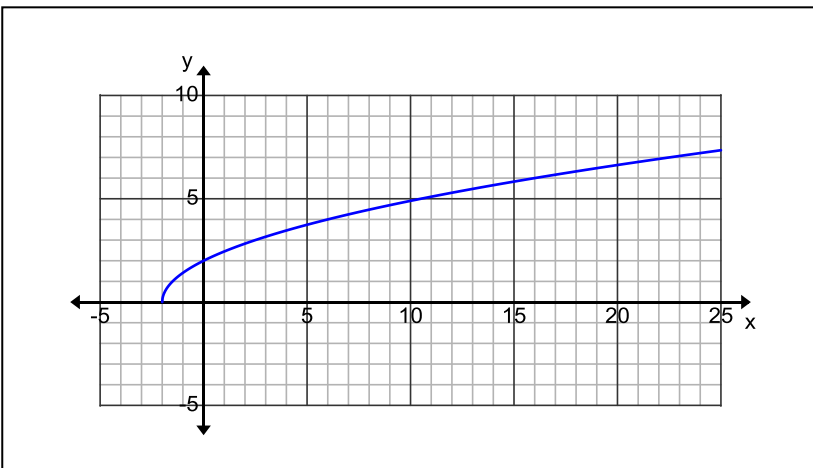
$$f(x) = \sqrt{x}$$

Notes/Observations about the domain and range of $f(x)$:



$$g(x) = \sqrt{x - 6}$$

Notes/Observations about the domain and range of $g(x)$:



$$h(x) = \sqrt{2x + 4}$$

Notes/Observations about the domain and range of $h(x)$:

Observations/Conjectures/Conclusions about the domains and ranges of radical (square root) functions: (Please be sure to explain and justify your statements.)

This page was intentionally left blank.

Growth Rates



Sisters Courtney and Nina have started an Internet business. Their business started out slow, but business is picking up. They want the business to double how much it earns in the next four months. Courtney and Nina know the company can grow at different rates. They investigate two growth rates, linear and exponential rates. Nina creates the following the following chart for the steady rate.

Linear Rate

\$2,000, _____ , _____ , _____ , \$4,000
January February March April May

1. Complete the sequence of earnings that grows at a linear rate.
Explain how you know it is linear.

Next the sisters consider the exponential rate. They create a similar chart.

Exponential Rate

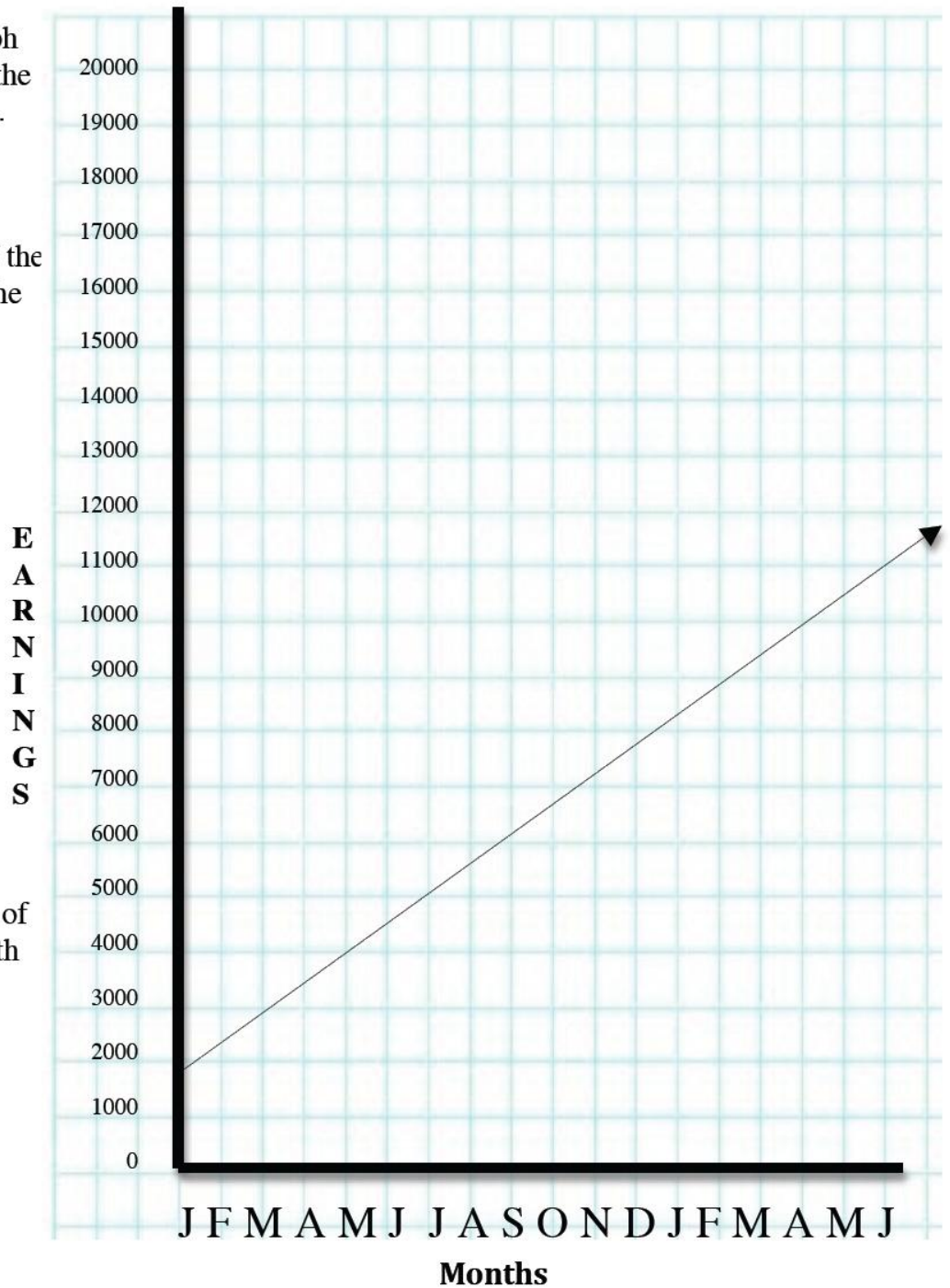
\$2,000, _____ , _____ , _____ , \$4,000
January February March April May

2. Complete the sequence of earnings that grow at an exponential rate.
Show how you determined the earnings for each month.

Courtney drew a graph of the earnings from the linear data calculated.

3. Draw the graph of the exponential data on the same axis.

4. Write an equation of the exponential growth rate.



5. They want to earn a million dollars. If the business grows at that exponential rate, in how many months would that occur? Show how you figured it out.

Functions Assessment

Name _____ Period ____

1. Match the equation to the graph.

$f(x) = 2x + 3$

$f(x) = 3 \cdot 2^x$

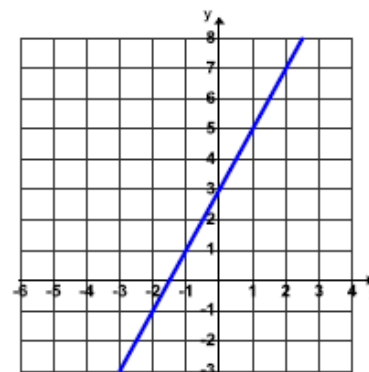
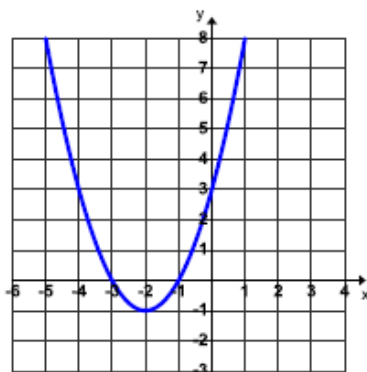
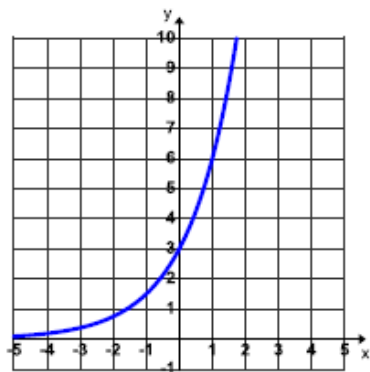
$f(x) = \sqrt{x} + 3$

$f(x) = 3x + 2$

$f(x) = x^2 + 4x + 3$

$f(x) = 2 \cdot 3^x$

$f(x) = x^3 + 3$



Equation: _____

Equation: _____

Equation: _____

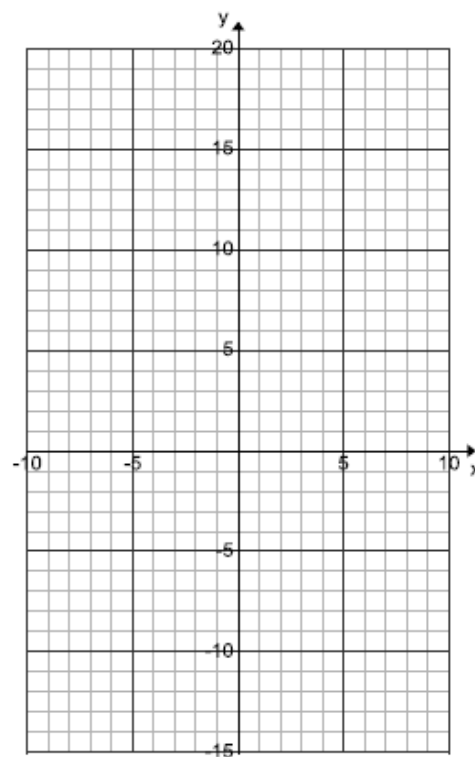
2. Write the equation for the following function.

x	f(x)
0	4
1	9
2	14
3	19
4	24

Equation: _____

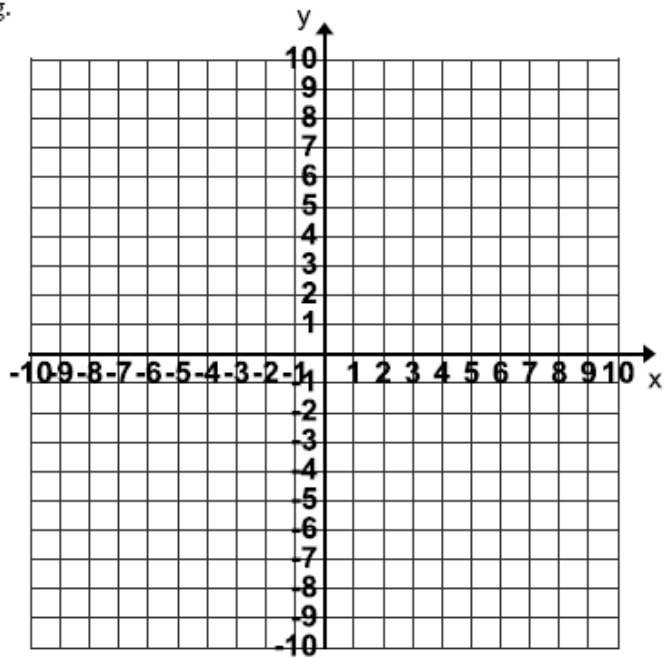
How did you know what kind of equation to use?

3. Graph the following function: $f(x) = x^3 + 1$



4. Given the function $f(x) = \sqrt{x}$, complete the following.

x	f(x)
0	
1	
4	
9	
16	



What is the domain of this function? Explain what this means.

What is the range of this function? Explain what this means.

5. Write the equation for the following situation: Louis starts with \$1 but doubles his money on the first day and continues to double his amount every day.

Equation: _____

How much money will Louis have on Day 5?

When will Louis have more than \$1000?

This page was intentionally left blank.

This page was intentionally left blank.